Location and Education in South African Cities under and after Apartheid

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We model a South African city during Apartheid (in which both schooling and mobility are restricted on the basis of race) and after Apartheid (in which no restrictions are imposed). We first show that the inequality between blacks and whites decreases when apartheid laws are removed. Indeed, blacks are better off because of human capital externalities due to the possibility of mixing with white students, whereas whites are worse off due to negative human capital externalities and intensified land market competition. After Apartheid, we also show that reducing the commuting costs of black children always increases the utility of black families and may even increase that of whites.

Key Words: Apartheid; South Africa; urban segregation; education externalities; urban land use.

1. INTRODUCTION

In June 1998, the South African newspaper the Star told the story of Nomakhazi Mdakane, a 46-year-old single mother who lives in the suburbs of Cape Town and commutes every day to a job located in a central part of the city. Up before dawn, she makes a 1-km walk to a taxi rank and often waits an hour before being able to board her only transport to work, at about 6 a.m. If she could afford it, she would buy a car, but half of her wages are spent on transport already and the rest goes to fees for her daughters, who attend an inner-city school.
Such stories are common in South African cities and illustrate some of the major problems that have been inherited from the past policy of Apartheid. Indeed, South Africa’s former segregation policy has had a tremendous impact on the workings of South African cities, leading to great inequalities in the distribution of income and human capital among spatially divided population groups. A key feature under Apartheid was that urban nonwhites were discriminated against in both their residential location and access to schooling; they were forced to reside in peripheral locations with segregated and low-quality education, far away from whites and central jobs.

The aim of this paper is to model the South African urban situation during and after Apartheid and to analyze its consequences in terms of education, housing, and mobility. It is indeed our contention that the mechanisms of school desegregation (after Apartheid) can only be fully understood at a local and spatial level. The main question is whether ending the restrictions on residential location and school choices will lead to more equality or larger disparities.

Keeping this issue in mind, we study two types of situations. In the Apartheid regime, whites live near the city center, where all jobs are located, whereas blacks (or nonwhites) are forced to reside in the outskirts of the city. Moreover, white children go to the centrally located white school, while black children attend a black peripheral school. Because of Apartheid, there is no competition between groups in education (black and white children do not mix, so there are no human capital externalities) or in the land market (there are two separate markets). There is, however, within-group competition.

In the post-Apartheid regime, there are no land-use or education restrictions so everyone can live or study anywhere in the city. We show that white families still reside in the vicinity of the city center and bid away black families to the outskirts of the city. This corresponds to the structure of many post-Apartheid South African cities (such as, for instance, Cape Town or Durban). In this context, as illustrated by the Star’s article, black parents still have long commuting trips to work in the city center, whereas whites have much shorter commuting trips. However, in the post-Apartheid regime, there is a certain amount of educational mixing, since some blacks are able to study at the central school (the former “white school”). If, for black students, there is an integration cost to study in a white school (because, for instance, they have to adapt to the English language or to the new norms), and if blacks are heterogeneous with respect to this cost, we show that, on average, those who attend the central school live relatively closer to the city center and have a low integration cost. The interesting feature of the post-Apartheid situation is that black families are induced to send their children to the central school, which leads to educational mixing.

We then compare these two regimes. For whites, it should be clear that they incur a utility loss because they face more competition in the land market (leading to higher land prices) and because their level of education decreases due to their mixing with black students (who inherited a lower initial human
capital). Concerning blacks (as a whole), there is a trade-off. On one hand, they gain because of human capital externalities, but on the other hand, they lose because of fiercer competition in the land market. We show that, as a net effect, black families are better off and inequality (as measured by the difference between the utilities of whites and blacks) decreases when restrictions are removed (as in the post-Apartheid regime).

The rest of the paper is as follows. The next section gives some stylized facts about South African cities that fit with our model. Section 3 describes our model, while Section 4 focuses on the Apartheid equilibrium. In Section 5, we determine the post-Apartheid equilibrium, and Section 6 compares the two types of equilibria. Section 7 concludes.

2. SOME FACTS ABOUT SOUTH AFRICAN CITIES

The aim of this section is to describe the South African situation under and after Apartheid. We focus on different elements that are relevant to our model, namely the structure of cities, transportation costs, inequality, and education.

2.1. The Structure of South African Cities

It is necessary to recall that Apartheid was implemented for almost half a century and resulted in tremendous disparities between communities, one of the main features being the land-use restrictions that were imposed on all communities. Under Apartheid, only whites could live close to the city center, where most jobs were located. The nonwhite labor force (i.e. “Asians/Indians,” “coloreds,” or “blacks/Africans,” according to the former racial classification established by the Apartheid regime) could live only in the periphery of cities, sometimes very far from the city center (see, e.g., Smith [27]). Racially homogeneous townships separated by buffer zones were created to prevent people from interacting with individuals from other communities. Typically, Asian and colored townships were distant but relatively closer to the city center, whereas black townships were located as far as possible from the center. Even though a certain amount of residential desegregation started to occur at the end of the 1980s, these spatial patterns of segregation still prevail in the 1990s. In Cape Town, for instance, the black–white index of dissimilarity was above 97% in 1991 (Christopher [12]) and still exceeded 93% in 1996.4 As Table 1 shows, there is a very high level of

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4 Considering two communities, blacks and whites, for example, the dissimilarity index is equal to

$$\frac{1}{2} \sum_i \left| \frac{\text{Blacks}_i}{\text{Blacks}} - \frac{\text{Whites}_i}{\text{Whites}} \right|,$$

where $i$ refers to neighborhoods. This index gives the percentage of individuals of a given type who would have to relocate to produce a homogeneous distribution of the population within the city. A dissimilarity index of less than 30% is considered low; between 30% and 60%, medium; and above 60%, high (see Cutler et al. [13]).
TABLE 1
Indices of Dissimilarity for the Five Major Metropolitan Areas in 1991

<table>
<thead>
<tr>
<th></th>
<th>Johannesburg</th>
<th>Cape Town</th>
<th>Durban</th>
<th>Pretoria</th>
<th>Port Elizabeth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asian-Black</td>
<td>94.5</td>
<td>98.0</td>
<td>90.9</td>
<td>91.5</td>
<td>99.2</td>
</tr>
<tr>
<td>Asian-Colored</td>
<td>88.6</td>
<td>94.0</td>
<td>89.1</td>
<td>96.4</td>
<td>82.1</td>
</tr>
<tr>
<td>Asian-White</td>
<td>90.0</td>
<td>90.6</td>
<td>98.3</td>
<td>96.3</td>
<td>93.4</td>
</tr>
<tr>
<td>Black-Colored</td>
<td>93.5</td>
<td>97.5</td>
<td>91.0</td>
<td>90.4</td>
<td>95.0</td>
</tr>
<tr>
<td>Black-White</td>
<td>89.6</td>
<td>97.3</td>
<td>66.6</td>
<td>87.1</td>
<td>98.2</td>
</tr>
<tr>
<td>Colored-White</td>
<td>93.9</td>
<td>96.3</td>
<td>94.7</td>
<td>92.4</td>
<td>97.7</td>
</tr>
</tbody>
</table>

Source: Christopher [12].

segregation in all South African metropolitan areas and between all communities. These values are extremely high since, even in highly segregated American cities such as Detroit, the dissimilarity index is “only” in the range of 70% (Cutler et al. [13]).

Not surprisingly, one of the main effects of racial zoning and segregation was to break down cities into very contrasted urban zones. In the Cape Metropolitan Area, for instance, an urban area that expands beyond 25 km and encompasses over 2.5 million inhabitants, most centers of opportunity are clustered around one edge of the city—the central business district and its closed surroundings (Mail and Guardian [20]). Indeed, the job-rich central areas of the city contrast with the townships and the poor peripheral informal housing areas. The central areas comprise centers of employment laid out along corridors extending outward from the port and city center. They mainly host middle and higher income people. In contrast, townships are inhabited by middle to lower income people with poor access to activities and services, while peripheral informal housing areas mainly consist of high-density slums. Needless to say, the main problem caused by the layout of South African cities is obviously the separation of workplace and residence. In Cape Town, more than 80% of formal employment is located in the CBD or along the corridors whereas less than 40% of the population lives there (Cape Metropolitan Council [8, 9]). Therefore, these spatial patterns result in a considerable amount of commuting (Cape Times [10]; Naude and Crous [24]).

2.2. Travel to Work and the Corresponding Commuting Costs

Mainly because of their respective locations within cities, the different communities experience very distinct commuting patterns in terms of distances traveled, time costs, and transportation modes. These differences are very detrimental to nonwhites: in South African cities, the average commuting distance for blacks, over 15 km, is twice as long as for whites who travel less than 7 km to go to work (Vines Mikula Associates [36]). In 1990, a resident commuting to Cape Town’s city center from Khayelitsha (one of the city’s black
TABLE 2
Mode of Transport of Metropolitan Commuters in 1992

<table>
<thead>
<tr>
<th></th>
<th>Whites</th>
<th>Asians</th>
<th>Coloreds</th>
<th>Blacks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public transport</td>
<td>7%</td>
<td>33%</td>
<td>53%</td>
<td>79%</td>
</tr>
<tr>
<td>Bus</td>
<td>4%</td>
<td>22%</td>
<td>10%</td>
<td>20%</td>
</tr>
<tr>
<td>Taxi</td>
<td>—</td>
<td>8%</td>
<td>8%</td>
<td>46%</td>
</tr>
<tr>
<td>Train</td>
<td>3%</td>
<td>3%</td>
<td>35%</td>
<td>13%</td>
</tr>
<tr>
<td>Car</td>
<td>87%</td>
<td>57%</td>
<td>36%</td>
<td>9%</td>
</tr>
<tr>
<td>Walking</td>
<td>2%</td>
<td>4%</td>
<td>7%</td>
<td>11%</td>
</tr>
<tr>
<td>Other</td>
<td>4%</td>
<td>6%</td>
<td>4%</td>
<td>1%</td>
</tr>
<tr>
<td>Average monthly cost (Rands)*</td>
<td>221</td>
<td>134</td>
<td>101</td>
<td>67</td>
</tr>
<tr>
<td>Average travel time (mn)</td>
<td>25</td>
<td>36</td>
<td>44</td>
<td>51</td>
</tr>
</tbody>
</table>

*1 Rand = 0.1635 US dollar (July 9, 1999).
Source: Vines Mikula Associates [36].

townships) had to face 2 hours and 40 minutes of transport, excluding waits at connection points (Urban Problems Research Unit [33, 34]). Table 2 shows that, in 1992, 87% of whites and 57% of Asians used their cars to commute, whereas 79% of blacks and 53% of coloreds resorted to public transportation. Observe that the use of taxis was quite frequent, especially among black commuters (46%). In 1999, this is even more true for blacks but also for other non-white communities, and taxis are used by 65% of urban commuters (Cape Argus [7]).

2.3. Inequality and Education

The spatial patterns of South African cities have contributed to the creation and perpetuation of significant disparities between population groups, notably in terms of income and education. In Cape Town, a city in which there are almost as many whites as blacks (23% and 27% of the local population, respectively) and where coloreds are in the majority (about one-half of the local population), 69% of white households have a monthly income above 3500 Rands, whereas 74% of black households have an income less than 1500 Rands (see Table 3).

The stark disparities in income compare well with education imbalances in South African cities. For many years, in accordance with Apartheid’s logic, there was a separation of educational provision on the basis of race. Prior to the 1994 elections that marked the end of Apartheid, education was operated by departments organized by race and geographic location. The system was very unequal, the objective being to limit the post-school opportunities of nonwhite

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5A taxi or minibus-taxi is a cheap and quite unsafe means of transportation that can board up to 15 people. The wide use of taxis among blacks is a recent phenomenon that started in the 1980s as a response to the inadequate public transport from townships and informal settlement areas (Cape Argus [6]).

6Asians accounts for the rest of the city’s population (3%). See Cape Metropolitan Council [8].
TABLE 3
Estimated Monthly Household Income Distribution in Cape Town in 1995

<table>
<thead>
<tr>
<th>Income Range</th>
<th>Whites</th>
<th>Asians</th>
<th>Coloreds</th>
<th>Blacks</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–1500 Rands</td>
<td>12%</td>
<td>27%</td>
<td>36%</td>
<td>74%</td>
</tr>
<tr>
<td>1501–2500 Rands</td>
<td>9%</td>
<td>18%</td>
<td>21%</td>
<td>14%</td>
</tr>
<tr>
<td>2501–3500 Rands</td>
<td>10%</td>
<td>13%</td>
<td>15%</td>
<td>5%</td>
</tr>
<tr>
<td>Above 3500 Rands</td>
<td>69%</td>
<td>42%</td>
<td>27%</td>
<td>6%</td>
</tr>
</tbody>
</table>

*Source:* CSIR [21].

children, essentially to menial occupations (World Bank [38]). Table 4 shows the discrepancies in the quality of education that was delivered to the different communities. In the early 1990s, just before the end of Apartheid, while Africans accounted for 75% of the country’s population, they only received 47% of recurrent government expenditure on education. In short, for every 4 Rands spent on a white child, only 3 Rands were spent on an Asian child, 2 Rands on a colored child, and 1 Rand on a black child (Thomas [32]). Moreover, classes in black schools were overcrowded (there were on average 42 students per teacher), and only 14% of black students studied through the end of high school (grade 12), whereas as much as 88% of white children were able to graduate from high school.

In view of these striking figures, one should not be surprised at the resulting discrepancies in the levels of education across population groups. White and black schooling systems were “two limiting cases in terms of the opportunities that they afforded” (Fedderke *et al.* [15]), and, obviously, such a system has been strongly discriminating. If we compare with the United States there is the same gap in terms of schooling levels between blacks and whites born in the early 1970s in South Africa and those born in the 1930s in America (Thomas [32]).

In the context of Apartheid’s abolition, the key issue is whether school desegregation can significantly contribute to the reduction of human capital imbalances. It should be noted that, even though limited desegregation began in state

TABLE 4
Inequality in Education in South Africa in 1991

<table>
<thead>
<tr>
<th></th>
<th>Whites</th>
<th>Asians</th>
<th>Coloreds</th>
<th>Blacks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population share</td>
<td>13%</td>
<td>3%</td>
<td>9%</td>
<td>75%</td>
</tr>
<tr>
<td>Share of expenditure</td>
<td>34%</td>
<td>5%</td>
<td>14%</td>
<td>47%</td>
</tr>
<tr>
<td>Pupils per teacher</td>
<td>19</td>
<td>22</td>
<td>24</td>
<td>42</td>
</tr>
<tr>
<td>Academic survival (up to grade 12)</td>
<td>88%</td>
<td>53%</td>
<td>20%</td>
<td>14%</td>
</tr>
<tr>
<td>Average years of education (men)</td>
<td>9.5</td>
<td>7.9</td>
<td>5.5</td>
<td>3.9</td>
</tr>
</tbody>
</table>

*Sources:* Case and Deaton [11], Thomas [32], and World Bank [38].
schools in the 1980s and the 1990s, unrestricted formal desegregation was only decided in 1995 and still comes up against many difficulties. Sociologists have pointed out the particularities associated with school desegregation in South Africa, which boils down to “integrating a majority group into a privileged minority culture” (Penny et al. [25]). In particular, black children bear some of the costs of integration, because they have to adapt to the language (English or Afrikaans) or to new norms at school (Naidoo [23]; Vally [35], Zafar [39, 40]). In South African cities particularly, these social obstacles to integration also involve spatial considerations. Indeed, parents who care about education may wish to reside close to good (central) schools and thus may be willing to pay higher housing prices (Case and Deaton [11]). Those who cannot afford to move might send their children on long and expensive commuting trips, as is currently observed. If moving and commuting costs are prohibitive, then the neighborhood location of schools reinforces the polarization in education by limiting the exposure of pupils to the world beyond their immediate community and thus aggravates inequality in education (Smit and Hennessy [26]).

The facts presented in this section have stressed the crucial role of space in the understanding of patterns of education and inequality by race. Therefore, taking into account the inherited inequality, the problems of access to schooling and the workings of cities, the role of this paper is to gauge the conditions under which school integration will succeed in South African cities.

3. THE MODEL

The city is closed and linear, with absentee landlords. There are two centers. The first one, the Central Business and Educational District (CBED hereafter), is at the city center (taken as the origin of the line), whereas the other one, the Suburban Educational District (SED hereafter), is exactly at the city fringe. The CBED has all of the jobs (and thus all of the firms), as well as one big representative school, whereas the SED only has one big representative school and no jobs at all.

There is a continuum of families uniformly distributed along the linear city, and they all consume the same amount of land (normalized to 1). The density of residential land parcels is taken to be unity, so that there are exactly $x$ units of housing within a distance $x$ from the CBED. The families belong to two different types, nonwhites or blacks (type B) and whites (type W) whose respective

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7The inefficiency of segregation, the need for qualification, and the de facto emergence of “gray areas” (mixed neighborhoods) boosted desegregation in schools even before the abolition of Apartheid laws. For an interesting presentation of school desegregation and subsequent issues in South Africa, see Naidoo [22, 23], Tikly [30, 31], and Penny et al. [25].

8All of these assumptions are very standard in urban economics, and relaxing them does not alter our main conclusions (see Fujita [16]).

9Observe that our model could incorporate informal employment since, in South African cities, a very large percentage of informal jobs are located in the city center.
masses are given by \( \bar{N}_B \) and \( \bar{N}_W \) (with \( \bar{N}_B + \bar{N}_W = \bar{N} \)). The only difference between these two types lies in the initial human capital endowments of adults, which is lower for blacks, i.e., \( \bar{h}_B < \bar{h}_W \), where \( \bar{h}_i (i = B, W) \) is the initial amount of human capital of a type \( i \) parent. This assumption reflects the fact that, because of South Africa’s history, levels of human capital between blacks and whites are unequal (see Table 4 and the discussion in Section 2.3).

A family consists of one parent and one child. We use a static model which captures our main message about location and educational choices. Parents are working while children are studying.

After Apartheid, a certain amount of integration is bound to take place within schools. We assume that there is a cost \( \theta \) to integration (see, e.g., Akerlof [1] for a discussion of peer pressures in black communities facing integration). In South Africa, this cost reflects the fact that it is the nonwhite learner who must adjust to the school norms, developing, for instance, competency in the English language (see Zafar [39] and our discussion at the end of Section 2.3). Observe that white children, being the reference group in the central school, do not incur any interaction cost with black children other than human capital externalities. We thus assume that only black children bear the cost of integration when they attend the central former white school. Moreover, not all black children have the same ability to integrate and interact with the other group. Formally, the integration cost, denoted by \( \theta \), is uniformly distributed on \([0, 1]\) among the group of black children.

The timing of the model is as follows. In stage 1, families choose their location in the city. Then, in stage 2, they decide which school to send their children to after the revelation of their integration cost.\(^{10}\) In stage 3, children participate in the educational process, and, in stage 4, families consume.

4. THE EQUILIBRIUM DURING APARTHEID

In this section, we develop a model where blacks are discriminated against in both the housing market and access to the central school. The first hypothesis means that blacks cannot reside in central locations near the CBED, a portion of the urban space exclusively reserved for whites. It is indeed a well-documented fact that under the laws of Apartheid white locations were central, whereas those of nonwhites were peripheral (see Smith [27] and our discussion in Section 2.1). The second assumption is in accordance with the first one since under Apartheid blacks did not have access to white (central) schools.

In this context, all workers (black or white parents) commute to the CBED, while white students travel to the CBED and black students travel to the SED. Families endogenously decide on their optimal residence within their allocated

\(^{10}\)Note that under Apartheid, there is no such choice since black and white children are forced to attend separate schools. The fact that the integration cost is revealed after the location decision is discussed at the beginning of Section 5.
group area, i.e., between the CBED (normalized to zero) and $\bar{N}_W$ for whites, and between $\bar{N}_W$ and the city fringe $\hat{x}_i \equiv \bar{N}$ for blacks. As a result, there is no competition for land between blacks and whites, but families compete within their respective group areas.

Since land consumption is normalized to 1, each parent located at a distance $x$ from the CBED has the following (realized) utility function and budget constraint:

$$u_i = z_i + \beta h_i, \quad i = B, W$$

$$\bar{y}_i = z_i + t_i x + \tau_i |x - \hat{x}_i| + R(x), \quad i = B, W$$

where $\beta$ is a positive parameter, $z_i$ denotes the consumption of the non-spatial composite good (whose price is taken as the numeraire) of a type $i$ family, $h_i$ is the human capital level of a type $i$ child, $\bar{y}_i$ is the income of a type $i$ parent, $t_i$ and $\tau_i$ are the respective transport costs per unit of distance for a type $i$ parent and a type $i$ child, and $\hat{x}_i$ is the location of the school so that, under Apartheid, we have $\hat{x}_W = 0$ and $\hat{x}_B = \bar{N}$. Finally, $R(x)$ denotes the equilibrium land rent at a distance $x$ from the CBED.

The following comments are in order. First, Eq (1) assumes that parents care about the human capital of their children ($\beta$ represents the parents’ degree of altruism), which affects their utility positively. Parents are thus altruists, and children’s levels of human capital depend on parental choices, as, for example, in Glomm [17]. The main difference with the standard human capital literature is that the present model does not only focus on explicit human capital investment but also on location choices, as in Benabou [2, 3], to the extent that location choices interact with educational choices. If there were no restrictions on location and educational choices, the preference for a school would depend on a trade-off between quality and proximity. We will elaborate further on this in the next section when we explore the unrestricted market equilibrium.

Second, we assume that the total unit commuting costs of blacks are lower than those of whites (i.e., $t_W + \tau_W < t_B + \tau_B$) and that black parents have higher unit transport costs than their children (i.e., $t_B > \tau_B$). The first assumption is easy to justify since blacks have a lower human capital and income than whites, and thus a lower opportunity cost of time, which in turn implies lower generalized unit commuting costs. This justification is also related to the fact that, in general, blacks and whites do not use the same transport modes (see Table 2). Whites mainly use cars, whereas blacks resort to public transportation, which, in the South African context, means that whites face higher monetary commuting costs per unit of distance than blacks.\(^{11}\) The second assumption is

\(^{11}\)In Cape Town, for instance, a recent study [5] evaluates the per kilometer commuting cost of cars at 1.52 Rands, while for public transportation (train, minibus taxi, or bus) it is less than 0.15 Rands.
also easy to understand since many black children use public transportation or
even walk to school. Therefore, if commuting cost is related to the parents’
opportunity cost of time, then it can be argued that transporting black children
is cheaper than transporting black adults because no parental time is used.

Third, a child’s level of human capital is determined by the quality of the
school he/she attends. The general educational output \( h_j \) of a school \( j \) \( (j = C, S, \text{ where } C \text{ stands for central school and } S \text{ for suburban school}) \) is given by

\[
h_j = \frac{N_{wj}}{N_{wj} + N_{bj}} \bar{h}_W + \frac{N_{bj}}{N_{wj} + N_{bj}} \bar{h}_B,
\]

(3)

where \( N_{ij} \) is the number of type \( i \) children \( (N_{ic} + N_{is} \equiv \bar{N}_i, \ i = B, W) \) in
school \( j \) \( (j = C, S) \), and \( \bar{h}_i \) \( (i = B, W) \) is the contribution of a type \( i \) child
\( (i = B, W) \) to the educational process, as measured by his/her parent’s inherited
level of human capital. The following comments are in order. First, Eq (3)
takes into account human capital externalities or spillovers, so that each student,
regardless of his/her ethnic origin, acquires the school’s average human capital
contribution. In other words, education is determined by the peer group effect
only.\(^\text{12}\) Second, the linearity of (3) implies that there are no aggregate gains or
losses from the mixing of children with different abilities. Third, each child’s
contribution is measured by his or her parent’s level of human capital \( \bar{h}_i \) (which
can be considered as a form of social capital that captures the quality of the
home learning environment). Fourth, education is not priced in our model. This
is quite different from the approach of Epple and Romano [14], who show that
when pupils have different contributions, it is optimal to subsidize the education
fees of high achievers because of spillover effects. Our framework does not
enable us to raise this issue because education is not priced. Observe, however,
that since \( \bar{h}_B < \bar{h}_W \), the mixing of blacks with white pupils (which will be
possible in the post-Apartheid regime) is always harmful to whites and beneficial
to blacks, so that even if education were priced competitively, some blacks
would still mix with whites, and this externality would still exist.

Under Apartheid, however, educational choices are restricted and there is no
interaction between blacks and whites at school, and thus there are no intergroup
education externalities. Then, by using the fact that under Apartheid, \( N_{WC} = \bar{N}_W \)
and \( N_{BS} = \bar{N}_B \), we have

\[
h_w \equiv h_C = \bar{h}_W
\]

(4)

\(^{12}\)Other, more general educational production functions have been used in the literature, all of
them putting a strong emphasis on the key role of peer group effects. See, for example, Summers
and Wolfe [29], Henderson et al. [19], or, more recently, the survey by Hanushek [18]. In the present
paper, the focus is on the interaction between education and location, and we have tried to keep the
model tractable by using a simple form of education production technology. Introducing educational
spending in our model would have complicated matters without altering our main message.
for white students and

\[ h_B \equiv h_S = \bar{h}_B \]  \hspace{1cm} (5) \]

for black students. This means that, under Apartheid, parents and children have the same human capital, so that this policy prevents the possibility of intergenerational improvement for blacks in terms of both location choices and human capital. In other words, under Apartheid, the parents’ education fully determines the schooling of their children (see, e.g., Wilson [37]).

By using (1) and (2), we obtain

\[ u_B = \bar{y}_B - t_B x - \tau_B (N - x) - R(x) + \beta \bar{h}_B, \hspace{0.5cm} x \in [N_W, \bar{N}] \]
\[ u_W = \bar{y}_W - (t_W + \tau_W) x - R(x) + \beta \bar{h}_W, \hspace{0.5cm} x \in [0, N_W]. \]

In this framework, the only choice of parents is to determine the family’s optimal location by maximizing their utility. Considering that, in equilibrium, all families of type \( i = B, W \) obtain the same utility levels \( v^A_B \) and \( v^A_W \) for blacks and whites, respectively, we are now able to write the families’ bid rents. They are equal to

\[ \Psi^A_B (x, v^A_B) = \bar{y}_B - t_B x - \tau_B (N - x) + \beta \bar{h}_B - v^A_B \]  \hspace{1cm} (6) \]

for blacks and

\[ \Psi^A_W (x, v^A_W) = \bar{y}_W - (t_W + \tau_W) x + \beta \bar{h}_W - v^A_W \]  \hspace{1cm} (7) \]

for whites. Moreover, we have

\[ \frac{\partial \Psi^A_B (x, v^A_B)}{\partial x} = -(t_B - \tau_B) < 0 \]
\[ \frac{\partial \Psi^A_W (x, v^A_W)}{\partial x} = -(t_W + \tau_W) < 0. \]

Our comments are the following. First, bid rents are always linear, and the trade-off is between land rents and commuting costs. Second, the bid rent of whites is always decreasing with distance from the CBED, while for blacks this is true only if \( t_B > \tau_B \), an assumption that we made and which is compatible with South African cities in which the commuting of black children involves little parental time (see our discussion above). Observe that both white parents and children commute to the city center whereas black parents commute to the city center while their children travel to the suburbs (see Fig. 1 for an illustration of the Apartheid city).

By normalizing the agricultural land rent (outside the city) to zero, we have the following definition:

\[ ^{13}\text{Bid rents are functions } \Psi_i (x, v) \text{ defined as the maximum rent that a family of type } i \text{ would be willing to pay at a given location } x \text{ so as to reach a given level of utility } v. \]
\[ ^{14}\text{All variables with superscript A refer to Apartheid.} \]
Definition 1. An Apartheid equilibrium (AE) is a triple \((v_A^A, v_W^A, R^A(x))\) such that

\[
R^A(x) = \begin{cases} 
\Psi_W^A(x, v_W^A) & \text{for } 0 \leq x \leq \bar{N}_W \\
\Psi_B^A(x, v_B^A) & \text{for } \bar{N}_W < x \leq \bar{N} \\
0 & \text{for } x > \bar{N}
\end{cases}
\]

(8)

\[
\Psi_W^A(\bar{N}_W, v_W^A) = 0 \tag{9}
\]

\[
\Psi_B^A(\bar{N}, v_B^A) = 0. \tag{10}
\]

By solving (9) and (10), we easily obtain the following equilibrium utilities for blacks and whites:

\[
v_B^A = \bar{y}_B - t_B \bar{N} + \beta \bar{h}_B \tag{11}
\]

\[
v_W^A = \bar{y}_W - (t_w + \tau_w)\bar{N}_W + \beta \bar{h}_W. \tag{12}
\]

The following comments are in order. First, in the Apartheid equilibrium, the two communities are totally separated so that there are two distinct housing markets (see Fig. 1). This urban configuration is typical of South African cities under Apartheid, as initially modeled by Brueckner [4]. Second, an increase in the income of one group raises its utility, whereas an increase in the transport cost decreases its utility. Moreover, the overall city size negatively affects black families, whereas white families are only affected by the size of their own community. In other words, in the Apartheid regime, whites are not affected by the presence of blacks. Last, in this context \(\tau_B\) does not affect the utility of blacks since in equilibrium we determine their utility at a location where the distance to school is zero. It is indeed easily verified in Fig. 1 that starting from
and moving inward, any black family experiences an increase in the child’s transport cost and a decrease in the parent’s commuting cost. Since \( t_B > \tau_B \) this causes a decrease in the family’s overall transport cost which is nevertheless exactly compensated for by a higher land rent.

The black–white inequality in this economy is equal to

\[
I^A \equiv v^A_W - v^A_B = \bar{y}_W - \bar{y}_B - (t_W + \tau_W)N_W + t_B N + \beta(\bar{h}_W - \bar{h}_B),
\]

so that

**Proposition 1.** In the Apartheid urban equilibrium, utilities and inequality are respectively given by (11)–(13). Inequality increases with

- the difference in human capital between black and white children,
- the difference in the income of parents, and
- the black parent’s unit commuting cost.

Inequality decreases with the commuting costs of white parents and white children.

5. THE EQUILIBRIUM AFTER APARTHEID

Since the end of Apartheid, there have been no spatial or educational restrictions (due to the removal of Apartheid laws), and workers commute to the CBED, while students can travel to the CBED or to the SED. Families endogenously choose between the CBED and the city fringe for their optimal residence \( \bar{x}_f = \bar{N} \) and choose the school attended by their child. Because this is truly the case in post-Apartheid South African cities, we assume that white parents do not send their children to the formerly black school located in the township. Different reasons can be given to justify this assumption. The more realistic one is that white parents are afraid of the high level of crime in black areas and are therefore very reluctant to send their children there.15

We also assume that schools, in particular, the formerly white school, are not capacity constrained.16

15Instead of just assuming it, we could have easily found a condition ensuring that in equilibrium white pupils do not attend black schools. In the context of our model, this is because, as we will see below, after Apartheid, whites still reside close to the city center, so that going to the suburban black school implies both higher commuting costs and lower human capital. This assumption is very realistic in post-Apartheid South African cities, and we have adopted it for simplicity.

16This is consistent with the assumption that education is not priced in our model. It should be clear that if school sizes were constrained, and if households bid for spots in a manner similar to that of the formation of bid rents, it would then be quite conceivable that education in the post-Apartheid regime could be somewhat closer to education under Apartheid, even if there were some unused capacity in the central school.
It is important to recall the timing of the model. In the first stage, all families choose their location in the city without knowing the type $\theta$ of the child (the cost of integration incurred by a black child studying in a formerly white school) but anticipating (with rational expectations) the number of black children who will attend the central school. In other words, black families base their location decision on expected utility, anticipating the proportion of black children that will go to the central school. In the second stage, types are revealed and black families choose their school depending on their previously determined location. The assumption that types are revealed only after the location is chosen takes into account the relative inertia of the housing market compared to educational mobility. In stage 3, children obtain different human capital levels, and then, in stage 4, families consume the composite good. Therefore, one of the main differences between the Apartheid and the post-Apartheid equilibria is that, in the latter, the initial type of children matters, whereas in the former, children were not given the opportunity to make use of their ability to integrate.

5.1. Housing versus Education

In the post-Apartheid regime, the choice of a school for blacks depends on a trade-off between location, transport costs (distance to the school), and human capital externalities. Indeed, as mentioned in the previous section, we assume that there are local spillovers or peer group effects in the production of education in the sense that studying among students that have a high human capital contribution increases one’s education. In particular, since by assumption the inherited human capital of black parents $\bar{h}_B$ is lower than that of white parents $\bar{h}_W$, Eq. (3) implies that the quality of a school increases with the relative number of white students. This means that blacks have an incentive to send their children to the central formerly white school to benefit from human capital externalities. Therefore, we can consider two types of black families: those who send their child to the central school C (type BC), whose mass is given by $N_{BC}$, and families whose child remains in the suburban school S after Apartheid (type BS), whose mass is given by $N_{BS}$, $N_{BC} + N_{BS} \equiv N_B$. This implies that human capital outputs in each school are given by $h_{BS} \equiv h_S$ and $h_{BC} \equiv h_W \equiv h_C$. In this context, the utilities of whites and blacks of type BS are still given by (1). However, since only the black children who attend the central school incur an integration cost $\theta$, their families have the following (realized) post-Apartheid utility function:

$$u_{BC} = z_{BC} + \beta h_{BC} - \theta.$$  \hspace{1cm} (14)

Observe that the integration cost of black students takes the form of a disutility when they attend the formerly white school, so that this disutility is increasing in the type $\theta$. Note that $\theta$ can also be interpreted as an inverse index of personal learning abilities to the extent that it may be less difficult for a high-ability child...
to be efficient in an environment that he or she is not used to. Furthermore, the budget constraint of a family of type \(i = BS, BC, W\) is equal to

\[
\tilde{y}_i = z_i + t_i x + \tau_i |x - \hat{x}_i| + R(x),
\]

(15)

where school locations are given by \(\hat{x}_{BC} = \hat{x}_W = 0\) and \(\hat{x}_{BS} = N\), and transport costs remain unchanged, so that \(t_{BS} = t_{BC} = t_B\) and \(\tau_{BS} = \tau_{BC} = \tau_B\).

Since families decide to locate before knowing the number of black pupils that will attend the central school, they base their decision on their (rational) expectations of the human capital output of this school, denoted by \(h^e_C\). Using (3), we have

\[
h^e_C = \frac{N_W}{N_W + N^e_{BC}} \tilde{h}_W + \frac{N^e_{BC}}{N_W + N^e_{BC}}\bar{h}_B,
\]

(16)

where \(N^e_{BC}\) is the expected number of black children going to the central school. By using (1) and (15), we obtain the following expected utility for a white family located in \(x\):

\[
U_W = \tilde{y}_W - (t_B + \tau_B)x - R(x) + \beta h^e_C.
\]

(17)

To compute their expected utility, black families take into account the probability that they will send their child to the central or the peripheral school after the child’s type is revealed. They do not know their type when computing their expected utility. However, for a given type \(\theta\) and location \(x\), if they decide to send their child to the central school, they will obtain the following utility (using (14)):

\[
U_{BC} = \tilde{y}_B - (t_B + \tau_B)x - R(x) + \beta h^e_C - \theta.
\]

(18)

In contrast, if they choose to send their child to the suburban school after the revelation of their type, the child will not bear any integration cost, and using (1), the utility of the family will amount to

\[
U_{BS} = \tilde{y}_B - t_B x - \tau_B(\bar{N} - x) - R(x) + \beta h_S,
\]

(19)

where \(h_S = \bar{h}_B\) is the human capital output of the suburban school.

Therefore, a black family located at a distance \(x\) from the CBED will decide to send its child of type \(\theta\) to the central school if \(U_{BC}\) is greater than \(U_{BS}\). This is the case when \(\theta\) is smaller than a threshold value \(\bar{\theta}(x, N^e_{BC})\), making a black family located in \(x\) indifferent between sending its child to either one of the two schools. It is given by

\[
\bar{\theta}(x, N^e_{BC}) = \beta(h^e_C - h_S) - \tau_B(2x - \bar{N})
\]

\[
= \beta(\bar{h}_W - \bar{h}_B)\frac{N_W}{N_W + N^e_{BC}} - \tau_B(2x - \bar{N}).
\]

(20)
In particular, the incentives for a particular black family to send its child to the central school depend on the location of that family in the city. More precisely, everything else being equal, people living further away from (closer to) the CBED are less (more) likely to send their child to the central school. This is because, due to the presence of commuting costs, $\tilde{\theta}$ is a decreasing function of $x$, the distance to the CBED, so that children living far away need to have a very low $\theta$ to go to the central school, while those living closer to the CBED can have a higher $\theta$ and still go to the central school. Figure 2 illustrates this point: a child of a given type $\theta$ will decide (ex post) to go to the central school if his or her family lives close to the CBED ($x < \bar{x}$), whereas the same child of type $\theta$ will go to the suburban school if his or her family resides far away from the CBED ($x \geq \bar{x}$).

Moreover, simple comparative statics on formula (20) yields the following results:  

**Proposition 2.** Black families are induced to send their children to the central formerly white school if

- the inherited difference in human capital between blacks and whites is large,
- the expected number of black children attending the central school is low, or

17To compute the results of Proposition 2, we have held $N_{BC}$ constant when varying any generic variable. This means that, when a parameter varies (e.g., commuting costs), individuals do not change their expectations for the distribution (here the expected number of black pupils that go to the white school). In other words, they assume that this variation has no impact on the distribution.
the number of white students in the city is large.

Black families living relatively closer to (further from) the CBED have more incentives (fewer incentives) to send their children to the central school when the unit transport cost of black children is high.

Proof. See the Appendix.

This proposition summarizes important results that will help us later in the equilibrium analysis. In particular, the incentives to go to the central school negatively depend on the expected number of black children going to the central school, \( N_{BC} \). In other words, there is a negative group externality on education incentives since, when \( N_{BC} \) increases, the expected human capital output of the central school \( h_C \) decreases, which in turn reduces the incentives to go to the central school.

We further assume that the condition
\[
\frac{\tau_B N^2}{\bar{N}_W} < \beta (\bar{h}_W - \bar{h}_B) < 1 - \tau_B (\bar{N}_B - \bar{N}_W)
\]
(21)

is met, which guarantees that \( \bar{\theta}(x, N_{BC}) \) is always strictly interior, i.e., that \( \bar{\theta}(x, N_{BC}) \) is always between 0 and 1.\(^{18}\)

We are now able to determine the expected utility of a black family residing at a distance \( x \) from the CBED, before the revelation of its \( \bar{\theta} \). It is given by
\[
EU_{\bar{\theta}}(x) = \int_0^\beta U_{BC} d\theta + \int_\beta^1 U_{BS} d\theta \\
= \int_0^\beta [\bar{y}_B - (\bar{\tau}_B + \tau_B) x - R(x) + \beta h_C x - \theta] d\theta \\
+ \int_\beta^1 [\bar{y}_B - (\bar{\tau}_B x - \tau_B (\bar{N} - x) - R(x) + \beta \bar{h}_B] d\theta \\
= \bar{y}_B - \bar{\tau}_B x + \tau_B x(1 - 2\bar{\theta}) - \tau_B \bar{N}(1 - \bar{\theta}) - R(x) \\
+ \beta \left[ (\bar{h}_W - \bar{h}_B) \frac{\bar{\theta} \bar{N}_W + N_{BC}}{\bar{N}_W + N_{BC}} \right] - \int_0^\beta \theta d\theta,
\]
\(^{18}\)Condition (21) expresses the requirement that \( \bar{\theta}(x, N_{BC}) \in [0, 1] \) for all \( (x, N_{BC}) \in [\bar{N}_W, \bar{N}] \times [0, \bar{N}_W] \). This condition corresponds to the assumption that blacks still occupy peripheral locations after Apartheid as will be proved below (see Proposition 3). Condition (21) is quite intuitive since it means that the difference in inherited human capital between blacks and whites \( (\bar{h}_W - \bar{h}_B) \) must take a medium value. In other words, if \( \bar{h}_W - \bar{h}_B \) were very large, then some black families residing close to whites would always send their child to the central school, regardless of the value of their integration cost, since the \( \bar{\theta} \) associated with their locations would tend to be greater than 1. In contrast, a very low value of \( \bar{h}_W - \bar{h}_B \) would discourage some black students from residing in the city’s periphery from attending the white school, even if their integration cost turns out to be zero, because the \( \bar{\theta} \) associated with those locations would tend to be less than 0. Assuming that \( \bar{h}_W - \bar{h}_B \) takes a medium value thus ensures that, before the type is revealed, any black child in the city stands a chance to attend or not to attend the formerly white school.
where \( h^e_C \) is defined by (16). Observe that the location decision is based on this expected utility, which is computed before the revelation of the child’s type. After it is revealed, school attendance is decided but location will remain unchanged.

We now obtain the whites’ bid rent by inverting Eq. (17). We thus have

\[
\Psi^P_W(x, v^P_W) = \tilde{y}_W - (t_W + \tau_W)x + \beta h^e_C - v^P_W,
\]

(22)

where \( v^P_W \) is the equilibrium utility of white families. Moreover, by inverting the expected utility of blacks, we obtain their bid rent,

\[
\Psi^P_B(x, v^P_B) = \tilde{y}_B - t_Bx + \tau_Bx(1 - 2\tilde{\theta}(x)) - \tau_B \tilde{N}(1 - \tilde{\theta}(x)) - v^P_B
\]

\[
+ \beta \left[ (\bar{h}_W - \bar{h}_B)\tilde{\theta}(x) \frac{\tilde{N}_W}{\tilde{N}_W + \tilde{N}^e_{BC}} + \bar{h}_B \right] - \int_0^\phi \theta \, d\theta,
\]

(23)

where \( v^P_B \) is the equilibrium utility of blacks.

Let us now determine the location of all agents in the city (see Fig. 3). We have

**Proposition 3.** After Apartheid, the structure of the city remains the same as under Apartheid: black families reside in the outskirts of the city (between \( \bar{N}_W \) and \( \bar{N} \)), whereas white families locate in the vicinity of the city center (between 0 and \( \bar{N}_W \)).

**Proof.** See the Appendix.

\[19\]All variables with superscript P refer to the post-Apartheid equilibrium.
This result is quite interesting since, even after Apartheid, when blacks and whites are free to choose where to live, we obtain a residential equilibrium (Fig. 3) similar to that of the Apartheid one (Fig. 1): whites still live close to the city center, whereas blacks reside in peripheral locations. In our model, because of low unit commuting costs, blacks have a “natural” tendency to live on the periphery; hence Apartheid did not distort the natural location pattern. This urban configuration broadly corresponds to most South African post-Apartheid cities such as Cape Town or Durban. A notable exception is Johannesburg, since, after Apartheid, whites and jobs have abandoned the city center to relocate in the suburbs. Brueckner [4] addresses the location reversal in Johannesburg by assuming that blacks consume less land than whites so that, in that model, Apartheid suppresses a natural tendency of blacks to live at the center. In our view, since there has been a dramatic decentralization of jobs in Johannesburg, our model can also address the specific case of Johannesburg just by flipping the city and thus locating the new CBED in the suburbs. In this particular context, blacks reside close to the historical city center and whites in the suburbs. It should thus be clear that our results remain valid for a city such as Johannesburg, since what matters is the distance between communities and centers of opportunity.

Observe that, in our model, the main difference between the two equilibria (before and after Apartheid) is that, after Apartheid, the spatial layout of the city is no longer imposed by law, but results from a bidding process between blacks and whites. As a consequence, competition in the land market is fiercer and land prices are higher after Apartheid than during Apartheid.

5.2. The Equilibrium

As stated above, once the location is decided, all types $\theta$ are revealed and expected values are observed (recall that all families have rational expectations). Therefore, we have

**Definition 2.** A post-Apartheid equilibrium (PAE) with rational expectations is a quadruple $(N_{BC}^P, v_W^P, v_B^P, R^P(x))$ such that

$$R^P(x) = \begin{cases} 
\Psi_W^P(x, v_W^P) & \text{for } 0 \leq x \leq N_w \\
\Psi_B^P(x, v_B^P) & \text{for } N_w < x \leq N \\
0 & \text{for } x > N
\end{cases}$$

$$\Psi_W^P(N_w, v_W^P) = \Psi_B^P(N_w, v_B^P) \quad (24)$$

$$\Psi_B^P(N, v_B^P) = 0 \quad (25)$$

$$N_{BC}^P = \int_{N_w}^{N} \theta(x, N_{BC}^P)dx. \quad (26)$$

20In the standard urban economics model, the lower the consumption of land, the steeper the bid rent.
Equation (24) indicates that, at the border \( \bar{N}_w \), the bid rents of blacks and whites are equal. Equation (25) states that the land rent paid at the city fringe is equal to the outside land rent normalized to 0. Equation (26) says that, under rational expectations, the expected number of black children going to the central school (denoted in equilibrium by \( N_{BC}^p \)) has to be the average number of black children that attend the central school. If we denote by \( \hat{\theta} \) the equilibrium proportion of black children attending the central school (i.e., \( \hat{\theta} \equiv N_{BC}^p/\bar{N}_B \)), Eq. (26) is equivalent to

\[
\hat{\theta} = \frac{1}{\bar{N} - \bar{N}_w} \int_{\bar{N}_w}^{\bar{N}} \hat{\theta}(x, N_{BC}^p)dx,
\]

where the right-hand side is the average proportion of black children whose \( \theta \) is lower than \( \hat{\theta}(x, N_{BC}^p) \) (when types are randomly distributed across space) and thus attend the central school.

We have

**Proposition 4.** Under condition (21), there exists a unique post-Apartheid equilibrium with rational expectations \( (N_{BC}^p, v_W^p, v_B^p, R(x)) \).

**Proof.** See the Appendix.

Observe that, in the first stage of our timing, each black family randomly chooses its location between \( \bar{N}_w \) and \( \bar{N} \) because, whatever the location, the expected utility level is the same and equals \( v_B^p \). It is only after the location has been decided that types are revealed and families choose their child’s school. In equilibrium, families who reside closer to the CBED are on average more likely to send their children to the central school, whereas those located closer to the city fringe \( \bar{N} \) tend to send their children to the suburban school (see our comments on formula (20) above). In other words, black families must stick to their initial location, which strongly affects the human capital attainment of their children.

We are now able to determine the equilibrium value \( N_{BC}^p \). By developing (26), we obtain the following second-degree equation:

\[
(N_{BC}^p)^2 + [(1 + \tau_B \bar{N}_B)\bar{N}_W] N_{BC}^p + \bar{N}_W \bar{N}_B [\tau_B \bar{N}_W - \beta (\bar{h}_W - \bar{h}_B)] = 0.
\]

There are two solutions to this equation, one of which is obviously negative, so that the other solution gives the value of \( N_{BC}^p \). Using (21), it is indeed easily verified that this solution is always between 0 and \( \bar{N}_B \).

In this context, there is a unique equilibrium \( \bar{\theta}^p(x) \) given by (using (20))

\[
\bar{\theta}^p(x) = \beta (\bar{h}_W - \bar{h}_B) \frac{\bar{N}_w}{\bar{N}_w + N_{BC}^p} - \tau_B (2x - \bar{N})
\]

and a unique equilibrium human capital output for the central school equal to

\[
h_C^p = \frac{\bar{N}_w \bar{h}_W + N_{BC}^p \bar{h}_B}{\bar{N}_w + N_{BC}^p},
\]

(27)
with
\[
\frac{\partial h_C^P}{\partial N_{BC}^P} = - \frac{(\bar{h}_W - \bar{h}_B) \bar{N}_W}{(\bar{N}_W + N_{BC}^P)^2} < 0. \tag{29}
\]

Equation (29) is quite intuitive since it means that when there are more black children who attend the central school, the educational outcome of this school decreases.

Now, since we know the equilibrium values of \(N_{BC}^P, h_C^P\) and \(\bar{\theta}(x)\), we can derive the equilibrium utility levels of blacks and whites. By using (27), (25) and (24), we obtain
\[
v_B^p = \bar{y}_B - t_B \bar{N} + \beta \bar{h}_B + \frac{1}{2} \left( \bar{\theta}(\bar{N}) \right)^2 \tag{30}
\]
and
\[
v_W^p = \bar{y}_W - (t_w + \tau_w) \bar{N}_W - (t_B - \tau_B) \bar{N}_B + 2 \tau_B \bar{N}_B \bar{N}_W
+ \beta \left[ (1 - 2 \tau_B \bar{N}_B)(\bar{h}_W - \bar{h}_B) \right] \frac{\bar{N}_W}{\bar{N}_W + N_{BC}^P} + \bar{h}_B. \tag{31}
\]

6. IMPLICATIONS FOR POST-APARTHEID POLICIES

In this section, we initiate a welfare analysis, derive some comparative statics results, and discuss the implications of education mixing. Let us first determine the changes in the utilities of blacks and whites and the resulting change in inequality between the two equilibria (i.e., during and after Apartheid). It is easily verified that, for blacks, we have
\[
\Delta v_B \equiv v_B^p - v_B^A = \frac{1}{2} \left( \bar{\theta}(\bar{N}) \right)^2 > 0. \tag{32}
\]
This is because, as a net effect, blacks benefit from the removal of restrictions in the land market and the educational system. Indeed, the average human capital of blacks always increases because of peer group effects, but land prices are higher due to fiercer competition for central locations. However, the first effect dominates the second one. For whites, we have
\[
\Delta v_W \equiv v_W^p - v_W^A = \Delta z_1 + \Delta z_2 + \beta \Delta h < 0, \tag{33}
\]
where
\[
\Delta z_1 = -(t_B - \tau_B) \bar{N}_B < 0
\]
\[
\Delta z_2 = 2 \tau_B \bar{N}_B \bar{N}_W \left[ t_B - \beta(\bar{h}_W - \bar{h}_B)/(\bar{N}_W + N_{BC}^P) \right] < 0
\]
\[
\Delta h = (h_C^P - \bar{h}_W) < 0.
\]
In this equation, $\Delta z_1$ is the net income loss for whites due to the entry of blacks in the competition for land, $\Delta z_2$ is the net income loss for whites due to the intensification of the competition for land caused by human capital externalities, and $\Delta h$ denotes the sheer human capital loss of white children. It is quite obvious why whites incur a loss in utility since land prices are higher and their human capital is lower ($h^P_C < \bar{h}_W$) after Apartheid. This can easily be seen by analyzing (33). The removal of land use restrictions negatively affects the net income of whites ($\Delta z_1 < 0$) due to fiercer competition in the land market. The removal of school segregation leads to a decrease in their human capital ($\Delta h < 0$) and has an indirect effect on the price of land (which increases) since the city center becomes more attractive for blacks who are attracted by the positive human capital externalities, which further reduces the net income of whites ($\Delta z_2 < 0$).

Let us denote by $I^P \equiv v^P_W - v^P_B$ the inequality after Apartheid. We have

$$\Delta I \equiv I^P - I^A = \Delta v_W - \Delta v_B < 0.$$  

The following result summarizes our discussion:

**Proposition 5.** When Apartheid is removed, whites are worse off whereas blacks are better off, and thus the inequality between blacks and whites decreases.

Let us now continue with the following result:

**Proposition 6.** After Apartheid, the equilibrium number of black students attending the central former white school increases when

- the initial human capital difference between blacks and whites is higher,
- the number of whites is larger, or
- their unit transportation cost is lower.

The human capital output of the central school varies in the opposite direction.

**Proof.** See the Appendix.

The results in Proposition 6 are quite intuitive and correspond to the inducement for blacks to go to the central school (see Proposition 2). In particular, in equilibrium, when the transport cost of black children is lower, the overall number of blacks attending the central school (the formerly white school) rises and the general level of human capital in the central school decreases.

Since the transport cost of black children $\tau_B$ is a key variable affecting $N^P_{BC}$, it is interesting to examine its impact on the utility levels of blacks and whites.

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21It is easily verified that, by using (21), $\tau_B < \beta(h_w - \bar{h}_B)/(N_w + N^P_{BC})$, so that $\Delta z_2$ is negative.
We have the following result:

**Proposition 7.** *In the post-Apartheid city, reducing the unit transport cost of black children increases the utility of blacks but has an ambiguous effect on the utility of whites. This means that a reduction in the unit transport cost of black children may possibly improve the utilities of both groups.*

**Proof.** See the Appendix.

For blacks, the intuition runs as follows. We can easily identify two opposite forces at work. On the one hand, when $\tau_B$ decreases, there is a direct and positive effect on the utility of blacks since the transport cost of black children becomes cheaper and accessibility is improved. On the other hand, this yields a negative group externality: when $\tau_B$ decreases, there is an indirect and negative effect on the utility of blacks since more (black) children attend the central school ($N_{BC}$ increases), which in turn tends to reduce utility (it is easily checked that $\partial v_B / \partial N_{BC} < 0$). As shown in the Appendix, the first effect is always dominant, so that the overall effect is positive.

For whites, when $\tau_B$ decreases, their utility tends to decrease since more black families are able to send their children to the central school, which reduces $h_C^P$ (the human capital output of the central school). However, this reduction leads to a weaker competition in the land market since central locations become less attractive for blacks and land rents decrease at the core of the city. This tends to increase the utility of whites. It is not clear which effect is dominant.

To complete our analysis, we now present some simulations that aim at highlighting the direction and magnitude of utility changes associated with the removal of Apartheid laws (since it is quite cumbersome to determine these analytically). In Table 5, we have reported the results of our analysis. In the base case (see the first column), 75% of the city’s residents are nonwhites and 25% are whites. The latter have three times as much inherited human capital as blacks. The latter have three times as much inherited human capital as blacks. When Apartheid is removed, the mean level of human capital for blacks increases by 14%. Indeed, among black children, 8.8% attend the central school, which enables them to raise their human capital by as much as 158% in comparison with what they would have obtained under Apartheid. Attending the formerly white school involves longer commuting trips and higher overall transport costs for black children, which increase by 4.7% on average. Of course, those black children who actually attend the central school face a much higher increase in their transportation costs.
TABLE 5
Simulation Analysis for Changes between the Apartheid and the post-Apartheid Equilibria (%)

<table>
<thead>
<tr>
<th></th>
<th>BASE</th>
<th>$\bar{\bar{N}}_B = 50%$</th>
<th>$\bar{\bar{N}}_W = 5\bar{\bar{h}}_B$</th>
<th>$\bar{\bar{N}}_W = 5\bar{\bar{h}}_W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N^\alpha_{BC}/\bar{\bar{N}}_B$</td>
<td>8.8</td>
<td>9.7</td>
<td>15.7</td>
<td>15.7</td>
</tr>
<tr>
<td>$\Delta h^\alpha_{BC}/\bar{\bar{h}}_B$</td>
<td>14.0</td>
<td>17.7</td>
<td>42.7</td>
<td>21.4</td>
</tr>
<tr>
<td>$\Delta h^\alpha_{BC}/\bar{\bar{h}}_W$</td>
<td>158.0</td>
<td>182.3</td>
<td>272.0</td>
<td>136.0</td>
</tr>
<tr>
<td>$\Delta 1h^\alpha_{BC}/\bar{\bar{h}}_B$</td>
<td>-14.0</td>
<td>-5.9</td>
<td>-25.6</td>
<td>-21.3</td>
</tr>
<tr>
<td>$\Delta 1h^\alpha_{BC}/\bar{\bar{h}}_W$</td>
<td>-14.9</td>
<td>-9.4</td>
<td>-23.3</td>
<td>-20.4</td>
</tr>
<tr>
<td>$\Delta R^\alpha/W^\alpha$ (landlord surplus)</td>
<td>-14.9</td>
<td>-9.4</td>
<td>-23.3</td>
<td>-20.4</td>
</tr>
<tr>
<td>$\Delta 1R^\alpha/W^\alpha$</td>
<td>-14.9</td>
<td>-9.4</td>
<td>-23.3</td>
<td>-20.4</td>
</tr>
<tr>
<td>$\Delta 1v^\alpha/W^\alpha$ (inequality)</td>
<td>-20.2</td>
<td>-14.4</td>
<td>-30.3</td>
<td>-30.3</td>
</tr>
<tr>
<td>$\Delta 1v^\alpha/W^\alpha$</td>
<td>-20.2</td>
<td>-14.4</td>
<td>-30.3</td>
<td>-30.3</td>
</tr>
<tr>
<td>$\Delta 1I^\alpha/I^\alpha$ (inequality)</td>
<td>-20.2</td>
<td>-14.4</td>
<td>-30.3</td>
<td>-30.3</td>
</tr>
</tbody>
</table>

Base case: $\beta = 1$, $\bar{\bar{t}}_B = 0.05$, $\tau_B = 0.025$, $\bar{\bar{r}}_W = 0.1$, $\tau_W = 0.075$, $\bar{\bar{h}}_B = 0.18$, $\bar{\bar{h}}_W = 0.06$, $\bar{\bar{\bar{y}}}_W = 0.18$, $\bar{\bar{\bar{y}}}_B = 0.06$, $\bar{\bar{\bar{N}}}_W = 0.25$, $\bar{\bar{\bar{N}}}_B = 0.75$. $\Delta$ expresses changes between the post-Apartheid and the Apartheid equilibria.

the increasing land rents they face (+100.9% on average in the white neighborhoods), is detrimental to their utility, which is reduced by 14.9%. Consequently, the inequality between blacks and whites decreases by 20.2%, but the surplus of the population, which we define as the weighted sum of the utilities of blacks and whites, decreases by 7.6%. In fact, to complete our surplus analysis, we must also take into account the utility of absentee landlords. Since utility (see (1) and (2)) is measured in units of consumption, we can add total utility to land rent. In this context, the overall surplus during and after Apartheid is defined by

$$S^k \equiv \bar{\bar{N}}_B v^k_B + \bar{\bar{N}}_W v^k_W + T L R^k, \quad k = A, P,$$

where $TLR^k$ denotes the equilibrium total land rent own by absentee landlords in regime $k = A, P$. From Table 5, we see that absentee landlords gain much from the removal of Apartheid since rents increase by an average 53.4% in the city as a whole, so that the total surplus $S$ is only reduced by 2.3%.23

In the second column of Table 5, we have changed the ethnic composition of the city by considering the case in which there are equal numbers of blacks and whites in the city. The key lesson is that, when the city accommodates a lower proportion of blacks, relatively more school integration takes place. This is both because blacks reside closer to the white school and because there is a

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23Observe that, when we compare the post-Apartheid equilibrium to the Apartheid equilibrium, the total surplus is always reduced because agents face integration costs and higher transportation costs while the total human capital in the economy remains constant. This result would not hold with a production function involving gains associated with heterogeneity.
large pool of white students, which is very attractive for blacks. Indeed, when blacks represent only 50% of the city’s population, 9.7% of black families send their children to the central school, and the gain in human capital for these children amounts to 182.3%. In spite of a greater proportion of blacks going to the central school, the proportion of black children within that school is small because whites are much more numerous than under the base case. Thus, the human capital loss of whites amounts to only 5.9%. Moreover, land competition is not as fierce in the white neighborhood, where land rents increase by only 34.1% on average, while it becomes a little fiercer in the black neighborhood, where rents increase by 18.6% on average. The overall effect is that landlords gain less, while the reductions in the population surplus and the total surplus are less dramatic. Inequality still significantly decreases.

In the third column, we show that larger inherited disparities in human capital ($\bar{h}_W = 5\bar{h}_B$) strongly induce black children to attend the central school (15.7%), where they can obtain a huge human capital gain (272%). This causes a significant reduction in inequality (−30.3%), which is nevertheless harmful to the total surplus (−5.7%). The last column of Table 5 presents quite similar effects when agents put twice as much weight on the human capital of their children than on composite good consumption ($\beta = 2$).

Whereas Table 5 focused on the possible gains resulting from the removal of Apartheid, Table 6 stresses the impact of a variation in the transport cost $\tau_B$ on post-Apartheid welfare. Starting from the base case used in Table 5, we consider the impact of a 20% increase in $\tau_B$ and a 20% decrease in $\tau_B$. First, observe that $\tau_B$ is negatively (positively) correlated with blacks’ (whites’) utility. This indicates that for whites, when $\tau_B$ decreases (see Eq. (39)), the negative effect (negative group externality) dominates the positive one (lower land rent). Moreover, when there is a twofold increase in $\tau_B$, 1.2% of black

<table>
<thead>
<tr>
<th>$\Delta v_B^p/v_B^p$</th>
<th>$\Delta v_W^p/v_W^p$</th>
<th>$\Delta N_{BC}/N_{PC}$</th>
<th>$\Delta I^p/I^p$</th>
<th>$\Delta P^S/P^S$</th>
<th>$\Delta S^p/S^p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.5</td>
<td>1.2</td>
<td>-1.2</td>
<td>1.9</td>
<td>0.5</td>
<td>-1.0</td>
</tr>
</tbody>
</table>

Base case: $\beta = 1$, $t_B = 0.05$, $\tau_B = 0.025$, $\tau_B = 0.1$, $t_B = 0.075$, $\bar{h}_W = 0.18$, $\bar{h}_B = 0.06$, $\bar{v}_W = 0.18$, $\bar{v}_B = 0.06$, $\bar{N}_W = 0.25$, $\bar{N}_B = 0.75$.
children attending the central school will shift to the suburban school. This leads to a higher population surplus (since the gains for whites outweigh the losses for blacks), but a lower total surplus (because of a deflating effect on land rents) and more inequality. In contrast, halving $\tau_B$ reduces both the population surplus and inequality while increasing the total surplus. This suggests a positive correlation between transport cost for black children and post-Apartheid inequality.

7. CONCLUSION

This paper is the first attempt to model education in a spatial urban context in South Africa under and after Apartheid. Although the model may seem quite stylized, we believe that it captures the basic features of most South African post-Apartheid cities. The key features of our model are (i) the historical difference in human capital, (ii) the fact that historically disadvantaged families have to pay a cost of adjustment to the school norms to attend high-quality schools and (iii) the difference in commuting costs. Observe that these factors are present in other countries, and our results could apply to contexts outside South Africa where minority students have been historically discriminated against (e.g., blacks in the United States).

Our results are the following. First, we obtain that after Apartheid, even though the land-use restrictions have been removed, the general structure of cities remains the same as under Apartheid: whites still reside close to the centers of opportunity, whereas blacks live far from these centers. Second, when Apartheid racial laws are removed, the inequality between blacks and whites can only decrease, because blacks benefit from local human capital externalities by mixing with white students, and because whites are worse off since both their human capital decreases and the land rent increases. Third, we explain the emergence of a new phenomenon widely recognized in post-Apartheid South Africa: the fact that some black students accept having to bear very long commutes to attend better schools located in white central locations.

Our model also has strong implications for policies that can be implemented in post-Apartheid South African cities, namely busing and transport to-school subsidies. Indeed, we have shown that a policy aimed at reducing the transport costs of black children has an ambiguous effect on the utility of whites but is always beneficial to blacks since it enables them to go to the best school in the city.

From the above discussion, it should be clear that space and education are central to the analysis of South African cities. Space and education were the two main criteria driving discrimination, since Apartheid was designed to prevent blacks and whites from interacting, especially in schools, and the spatial separation of communities was the most efficient way to ensure that no such interaction would ever happen. Our analysis shows that post-Apartheid integration can be promoted through education, even though the spatial separation between communities remains.
APPENDIX

Proof of Proposition 2. By differentiating (20), we easily obtain

\[ \frac{\partial \tilde{\theta}}{\partial (\bar{h}_W - \bar{h}_B)} > 0, \quad \frac{\partial \tilde{\theta}}{\partial \overline{N}_W} > 0, \quad \frac{\partial \tilde{\theta}}{\partial \bar{x}} < 0, \quad \frac{\partial \tilde{\theta}}{\partial \overline{N}^e_{BC}} < 0 \]

\[ \frac{\partial \tilde{\theta}}{\partial \tau_B} \geq 0 \iff x \leq \frac{\overline{N}}{2}. \]

The economic justifications for these effects are quite straightforward and are discussed in the core text of the paper. The number of white pupils and the difference in inherited human capital induce blacks to attend the white school because the larger this difference is or the higher is the number of whites, the greater the potential human capital gains for blacks. In contrast, the distance to the white school and the expected number of blacks attending the white school have a negative impact on the willingness of black children to attend the central school because of transport costs and the negative peer group externality. Finally, an increase in the unit transport cost of black children strengthens the families’ incentives to send their children to the closest school. So, since \( \overline{N}/2 \) is exactly the middle point between the two schools, when indifferent families reside on the right (on the left) of \( \overline{N}/2 \), a rise in \( \tau_B \) induces them to send their child to the suburban (central) school.

Proof of Proposition 3. In this proof, we calculate the bid rent slopes for whites and blacks, respectively, and show that whites have a steeper bid rent, so that, in equilibrium, whites do indeed bid blacks away from central locations.

From Eq. (22) we have

\[ \frac{\partial \Psi^W_P(x, v^p_W)}{\partial x} = -(t_W + \tau_W) < 0. \]

From Eq. (23), we can write

\[ \frac{\partial \Psi^B_P(x, v^p_B)}{\partial x} = -t_B + \tau_B \left[ 1 - 2\tilde{\theta}(x, N^e_{BC}) \right] + \frac{\partial \tilde{\theta}(x, N^e_{BC})}{\partial x} \left[ \beta (\bar{h}_W - \bar{h}_B) \frac{\overline{N}_W}{\overline{N}_W + \overline{N}^e_{BC}} + \frac{N^e_{BC} - \tau_B(2x - \overline{N}) - \tilde{\theta}(x, N^e_{BC})}{\bar{x}} \right]. \]

Since, by definition of \( \tilde{\theta} \), we have \( \tilde{\theta}(x, N^e_{BC}) = \beta (\bar{h}_W - \bar{h}_B) \overline{N}_W / (\overline{N}_W + N^e_{BC}) - \tau_B(2x - \overline{N}) \), the term in brackets cancels out, so that we have

\[ \frac{\partial \Psi^B_P(x, v^p_B)}{\partial x} = -t_B + \tau_B \left[ 1 - 2\tilde{\theta}(x, N^e_{BC}) \right]. \]
Since by assumption \( t_B > \tau_B \), then because \( \tilde{\theta} > 0 \), \( t_B > \tau_B \left[ 1 - 2 \tilde{\theta}(x, N_{BC}^e) \right] \), and thus the bid rent \( \Psi_B^e(x, v_B^p) \) is always downward sloping.

Furthermore, it is easy to verify that the bid rent of whites is always linear, whereas the bid rent of blacks is strictly convex, since

\[
\frac{\partial^2 \Psi_B^e(x, v_B^p)}{\partial x^2} = -2\tau_B \frac{\partial \tilde{\theta}(x, N_{BC}^e)}{\partial x} = 4\tau_B^2 > 0.
\]

The latter result means that, in absolute terms, the slope of blacks’ bid rent depends positively on \( \tilde{\theta}(x, N_{BC}^e) \). This is because the higher \( \tilde{\theta} \) is, the more attractive is the city center and thus the steeper is the bid rent. Finally, since we have assumed that \( t_W + \tau_W > t_B + \tau_B \) and since \( \tilde{\theta} > 0 \), it is easily checked that

\[
\forall x \in [0, \overline{N}], \quad \left| \frac{\partial \Psi_W^e(x, v_W^p)}{\partial x} \right| > \left| \frac{\partial \Psi_B^e(x, v_B^p)}{\partial x} \right|,
\]

so that whites always reside closer to the city center than blacks. In equilibrium, white families live between 0 and \( \overline{N}_W \), whereas black families locate between \( \overline{N}_W \) and \( \overline{N}_B \).

**Proof of Proposition 4.** First, we know that in standard urban economics models, an urban equilibrium always exists and is unique (see Fujita, 1989, Chap. 4, who shows that the existence and the uniqueness of an urban equilibrium are guaranteed as soon as bid rents can be ranked in order of relative steepness, here blacks and whites). Second, in our model, we also need to show that there exists a uniquely determined \( N_{BC}^p \) with \( 0 < N_{BC}^p < \overline{N}_B \). Let us consider the following function, which implicitly determines \( N_{BC}^p \):

\[
\Phi(N_{BC}^p) \equiv \int_{N_W}^{N_B} \tilde{\theta}(x, N_{BC}^p) dx - N_{BC}^p = \overline{N}_W \overline{N}_B \left[ \beta \frac{\bar{h}_W - \bar{h}_B}{N_W + N_{BC}^p} - \tau_B \right] - N_{BC}^p.
\]

(35)

It is easily verified that this continuous function is always decreasing in \( N_{BC}^p \) and that, by using (21), we have \( \Phi(0) > 0 \) and \( \Phi(\overline{N}_B) < 0 \), so that there always exists a unique \( N_{BC}^p \) between 0 and \( \overline{N}_B \). Consequently, there exists a unique equilibrium.

**Proof of Proposition 6.** Let us start with the comparative statics on \( N_{BC}^p \). By using (35), simple differentiation provides for any generic exogenous variable \( k \),

\[
\frac{\partial N_{BC}^p}{\partial k} = -\frac{\partial \Phi(N_{BC}^p)/\partial k}{\partial \Phi(N_{BC}^p)/\partial N_{BC}^p},
\]

which has the sign of \( \partial \Phi(N_{BC}^p)/\partial k \) since \( \partial \Phi(N_{BC}^p)/\partial N_{BC}^p < 0 \). Using the definition (35) of \( \Phi(N_{BC}^p) \), the comparative statics on \( N_{BC}^p \) follows immediately for parameters \( (\bar{h}_W - \bar{h}_B), \overline{N}_W, \) and \( \tau_B \).
We have
\[ \frac{\partial N^p_{BC}}{\partial (\bar{h}_w - \bar{h}_b)} > 0, \quad \frac{\partial N^p_{BC}}{\partial \bar{N}_w} > 0, \quad \frac{\partial N^p_{BC}}{\partial \tau_B} < 0. \]

Concerning the comparative statics on \( h^p_C \), we have
\[ \frac{\partial h^p_C}{\partial k} = \frac{\partial h^p_C}{\partial N^p_{BC}} \frac{\partial N^p_{BC}}{\partial k}, \]
so that, by using (29), we have
\[ \frac{\partial h^p_C}{\partial (\bar{h}_w - \bar{h}_b)} < 0, \quad \frac{\partial h^p_C}{\partial \bar{N}_w} < 0, \quad \frac{\partial h^p_C}{\partial \tau_B} > 0. \]

**Proof of Proposition 7.** By totally differentiating (30), we obtain
\[ \frac{\partial v^p_B}{\partial \tau_B} = -\bar{\theta} \left[ \frac{\partial N^p_{BC}}{\partial \tau_B} \frac{\beta (\bar{h}_w - \bar{h}_b) \bar{N}_w}{[\bar{N}_w + N^p_{BC}]^2 + \bar{N}} \right]. \]  

Applying (36) with \( k = \tau_B \), we have that
\[ \frac{\partial N^p_{BC}}{\partial \tau_B} = -\frac{\bar{N}_w \bar{N}_B}{(\beta (\bar{h}_w - \bar{h}_b) \bar{N}_w \bar{N}_B) / [\bar{N}_w + N^p_{BC}]^2 + 1} < 0. \]  

Finally, plugging (38) into (37), we obtain
\[ \frac{\partial v^p_B}{\partial \tau_B} = -\bar{\theta} \left[ \frac{\bar{N}_w \bar{N}_B^2 \beta (\bar{h}_w - \bar{h}_b) \bar{N}_w + (\bar{N}_w + N^p_{BC})^2}{\bar{N}_w \bar{N}_B \beta (\bar{h}_w - \bar{h}_b) + (\bar{N}_w + N^p_{BC})^2} \right] < 0. \]

Next, by differentiating (31), we have
\[ \frac{\partial v^p_W}{\partial \tau_B} = \bar{N}_B (1 + 4 \tau_B \bar{N}_w) - \frac{\beta (\bar{h}_w - \bar{h}_b) \bar{N}_w}{(\bar{N}_w + N^p_{BC})^2} \times \left[ \frac{\partial N^p_{BC}}{\partial \tau_B} \left( 1 - 2 \tau_B \bar{N}_B \right) + 2 \bar{N}_B (\bar{N}_w + N^p_{BC}) \right] \geq 0, \]

the sign of which is ambiguous since the first term is positive and the second term can be shown to be negative (using (38)).
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