

Paris School of Economics, M1 APE

Microeconomics 1, Problem Set 1

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Exercise 1

1. Is the function $u(x_1, x_2) = x_1^{1/3} x_2^{1/3}$ concave over its domain?
2. Let $u(x_1, x_2) = x_1 x_2$. Draw the indifference curve defined by $u(x_1, x_2) = 4$. Compute the marginal rates of substitution of good 1 for good 2 at the bundles $(1, 4)$ and $(2, 2)$.
3. The utility function u defined in question 2 is now composed by a C^2 strictly increasing function v . Show that this transformation has no effect on the marginal rates of substitution.
4. Consider two individuals, called 1 and 2, and assume that $MRS_{1,2}^1(x_1, x_2) \neq MRS_{1,2}^2(x_1, x_2)$. Explain why each of them can benefit from an exchange.

Exercise 2

Consider an agent, allowed to consume two goods 1 and 2. For each of the preferences described below: (a) propose a utility function representing them and draw the indifference curves in the Cartesian space (x_1, x_2) ; (b) calculate the marginal rate of substitution of good 1 for good 2 and interpret; (c) examine whether preferences are convex.

1. Good 1 has no effect on the welfare of Marie-Ange: she is indifferent between consuming it or not and she does not care about the quantity she uses. On the contrary, she always prefers having more good 2.
2. Marion prefers (x_1, x_2) to (y_1, y_2) if and only if $\min(x_1, x_2) \geq \min(y_1, y_2)$.
3. Maria is indifferent between the goods: she only cares about the total amount of good she has.
4. Katarina prefers (x_1, x_2) to (y_1, y_2) if and only if $x_1^2 + x_2^2 \geq y_1^2 + y_2^2$.
5. Malika prefers (x_1, x_2) to (y_1, y_2) if and only if $x_1 x_2 \geq y_1 y_2$.
6. Heather prefers (x_1, x_2) to (y_1, y_2) if and only if $\ln x_1 + \ln x_2 \geq \ln y_1 + \ln y_2$.

Exercise 3

1. Let \succsim be strongly monotone. Show that it is monotone.
2. Let \succsim be monotone. Show that it is locally non-satiated.
3. Draw a convex preference relation which is locally non-satiated but is non-monotone.
4. Exhibit an example of a preference relation which is not continuous but is representable by a utility function.

Exercise 4

Let us consider a consumer in a two-good economy. His consumptions of goods 1 and 2 are denoted x_1 and x_2 respectively. His tastes are represented by a preference relation \succsim defined over the consumption vectors $(x_1, x_2) \in \mathbb{R}_+ \times \mathbb{R}_+$. The indifference relation associated with \succsim is denoted \sim . The following assumptions are made:

- A1. \succsim is complete;
 - A2. $(\tilde{x}_1, 0) \succsim (x_1, 0)$ if and only if $\tilde{x}_1 \geq x_1$;
 - A3. all indifference curves are linear in the (x_1, x_2) -space. The indifference curve through $(\tilde{x}_1, 0)$ as equation $x_1 + 2x_2\sqrt{\tilde{x}_1} = \tilde{x}_1$.
1. Construct a utility function which represents the consumer's preferences.
 2. Compute the marginal rate of substitution of good 1 for good 2.

Exercise 5

You are given the following information about consumer's consumption plans in three scenarios:

Price/litre	Scenario 1	Scenario 2	Scenario 3
Orange	2	3	2.5
Pineapple	3	2	2.5

1. A first consumer claims that, in the first scenario, he would buy 10 litres orange juice and 10 litres pineapple juice; in the second scenario, he would buy 11 litres orange juice and 9 litres pineapple juice. Is it possible to represent his choices through a rational preference relation over consumption bundles?
2. A second consumer claims that, in the first scenario, he would buy 15 litres orange juice and 15 litres pineapple juice; in the second scenario, he would buy 16 litres orange juice and 14 litres pineapple juice; in the third scenario, he would buy 13 litres orange juice and 18 litres pineapple juice. Show that this individual's preferences as revealed by his choices cannot be transitive.