Paris School of Economics, Master 1 APE
Microeconomics 1, Problem Set 3

Michaelmas Term 2007-2008

Exercise 1

Part I:

Consider a consumer who consumes l goods, $l = 1,...,L$. His consumption set is: $X = \prod_{l=1}^{L} [a_l, +\infty[$, where $a_l > 0$ for each l. The consumer's preferences over the consumption bundles $x = (x_l)_{l=1,...,L}$ are represented by the utility function:

$$u(x_1, x_2) = k \prod_{l=1}^{L} (x_l - a_l)^{\alpha_l}, \tag{1}$$

with $k > 0$, $\alpha_l > 0$ for $l = 1,...,L$ and $\sum_{l=1}^{L} \alpha_l = 1$. Let $p = (p_l)_{l=1,...,L}$ be the price vector.

1. Give an economic interpretation of $a_l > 0$, $l = 1,...,L$.
2. *From now on*, it is assumed that $w > \sum_{l=1}^{L} p_i a_i$, where w is the consumer's wealth. Why is this assumption introduced?
3. Derive the Marshallian demands for goods l, $x_l(p,w)$. Check that it is homogenous of degree zero in (p,w) and that is satisfies Walras' law. Are there inferior goods? Giffen goods?
4. Derive the indirect utility function.
5. Derive the expenditure function $e(p,\overline{u})$ and the Hicksian demand function $h(p,\overline{u})$. Check that the former is increasing in each of its arguments.
6. Is any good substitute for or complement to another?

Part II:

Now $L = 2$. You are told the expenditure function was $e(p,\overline{u})$ derived in Question I.5:

$$e(p,\overline{u}) = a_1 p_1 + a_2 p_2 + e^{\overline{u}} \delta^{-\delta} (1-\delta)^{-(1-\delta)} p_1^{\delta} p_2^{1-\delta}, \tag{2}$$

where $\delta = \alpha_1 / (\alpha_1 + \alpha_2)$. You are given no further information.

1. Invert $e(p,\overline{u})$ to find the indirect utility function.
2. Use Roy's identity to find the Marshallian demand function $x(p,w)$.
3. Find the Hicksian demand function $h(p,\overline{u})$.
4. Confirm Shepard's Lemma holds.
5. Check that your answers coincide with those of Part I for $L=2$, $a_i=0\ \forall i$, and $k=1$.

Exercise 2

John earns £5 pounds per hour. He has 100 hours per week which he can use for either labour or leisure. The government institutes a plan in which each worker receives a £100 grant from the government, but has to pay 50% of his or her labour income in taxes. If his utility function is

$$U(c,L)=cL, \tag{3}$$

where c is the amount spent to consume goods and L hours of leisure per week, how many hours per week will John choose to work?

Exercise 3

It is often claimed that wages should increase at the same rate as the "cost of living". This exercise aims at precisely defining the cost of living and would like to cast light on the effects the wage policy has on individual welfare.

For this purpose, let us consider a consumer whose utility function is $U(x)$, where $x\in\mathbb{R}_+^n$ is a consumption bundle. Let R be his exogenous income. U is assumed to be quasiconcave and differentiable with $\partial U(x)/\partial x_i>0$, $\forall i=1,\ldots,n$.

1. The index of increase in the cost of living between p^0 and p^1 is defined as the ratio $I=\frac{e(p^1,u^0)}{e(p^0,u^0)}$, with $u^0=V\left(p^0,R\right)$. Explain why.
2. Let $x^0=x\left(p^0,R\right)$, $x^1=x\left(p^1,R\right)$. The Laspeyres index L is defined as $L=\frac{p^1x^0}{p^0x^0}$ and the Paasche index as $P=\frac{p^1x^1}{p^0x^1}$.
   (a) What is the economic interpretation of these indices?
   (b) Show that $L=\sum_{i=1}^n\frac{p_i^0x_i^0}{p^0x^0}\frac{p_i^1}{p_i^0}$. Interpretation?
   (c) Show that $L\geq I$. Establish that if the income of the consumer is indexed to L (i.e. $R^1=L.R^0$, with $R^t=$income at t), then the consumer's utility increases.
3. Let $I_r=\frac{e(p^1,u^1)}{e(p^0,u^1)}$, with $u^1=V\left(p^1,R\right)$.

(a) Interpret I_r.

(b) Show that $P \leq I_r$.

(c) What would happen if the consumer's income were indexed to P?

Exercise 4

A well-behaved consumer lives in a three-good economy (goods A, B and C), faces prices (p_A, p_B, p_C) and has income w. His demand functions for A and B are given by:

$$A = \alpha_0 + \alpha_1 \frac{p_A}{p_C} + \alpha_2 \frac{p_B}{p_C} + \alpha_3 \frac{w}{p_C}, \quad (4)$$

$$B = \beta_0 + \beta_1 \frac{p_A}{p_C} + \beta_2 \frac{p_B}{p_C} + \beta_3 \frac{w}{p_C}. \quad (5)$$

1. Explain how the demand function for C can be obtained.
2. Are (4) and (5) appropriately homogeneous?
3. At one set of prices and income, $A = 1$ and $B = 2$. At another set of prices, $A = 1$ and $B = 1$. What is the income elasticity of demand for A?