Paris School of Economics, Master 1 APE
Microeconomics 1, Problem Set 4

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Exercise 1

Consider any consumer whose demand functions for the only two goods in the economy, 1 and 2, read:

$$x_1(p_1, p_2, w) = \alpha \frac{w}{p_1}, \quad (1)$$

$$x_2(p_1, p_2, w) = (1-\alpha) \frac{w}{p_2}. \quad (2)$$

Our aim is to find a utility function which generates these demand functions.

1. Check that these functions are homogenous, that they satisfy Walras' law and that the Slutsky matrix is negative semi-definite.
2. To recover a utility function U which generates $x_1(p_1, p_2, w)$ and $x_2(p_1, p_2, w)$, one can proceed as follows:
   (a) Use homogeneity to normalize w to 1 and express each price as a function of $\alpha, x_1(p_1, p_2, w)$, $x_2(p_1, p_2, w)$.
   (b) Consider any indifference curve $U(x_1, x_2) = K$. For each given K, write $x_2 = \phi(x_1)$. Derive $d\phi(x_1)/dx_1$ and solve this differential equation.
   (c) What is the corresponding utility function?

Exercise 2

One is interested in the choice of the married women whether or not to work. This decision depends on the husband's wage, on the wage the wife can earn, on the time available to her, and on the value of her leisure time.

Each married woman will choose to work if the wage she is offered is above the minimum subjective value she places on her loss of leisure.

Part I:

Let us consider the following model. Miss Z has a utility function which depends on the consumption of a composite her consumption x, whose price is denoted p, and leisure L, measured in hours:

$$U(x, L) = \alpha \log L + (1-\alpha) \log x. \quad (3)$$

Her exogenous wealth is R_0 and she can divide T_0 hours between labour and leisure. The hourly wage at which she can be hired is w.

1. Show that her budget constraint reads:

$$px + wL = R_0 + wT_0. \tag{4}$$

   Interpretation?

2. Determine her Marshallian demand for consumption and leisure and thus her labour supply.

3. Check that Miss Z won't be willing to work when w is less than a threshold value $\overline{w}$, called her *reservation wage*. How does this threshold depend on α, R_0, and T_0?

4. Derive Miss Z indirect utility function $V(w, T_0, R_0)$.

5. The value Miss Z places on time is defined as:

$$V' = \frac{\frac{\partial V(w,T_0,R_0)}{\partial T_0}}{\frac{\partial V(w,T_0,R_0)}{\partial R_0}}. \tag{5}$$

   Notice the analogy between V' and the MRS to interpret the former. Check that $V' = \max\{w, \overline{w}\}$.

Part II:

We now focus on the consumption-leisure trade-off for any well-behaved utility function $U(x, L)$. The budget constraint remains the same as in Part I.

1. Use the Slutsky equation to express the changes in L and x when w varies.

2. Show that if x is a normal good, consumption increases when w increases. Interpretation? (In particular note that the income effect is always positive). How does L change when w increases?

Exercise 3

A consumer has utility function in the form:

$$U(bx_1, x_2, ..., x_L), \tag{6}$$

where $b > 0$ is a parameter. (An increase in b can be interpreted as making the consumer more efficient at enjoying commodity x_1).

The elasticity of demand for commodity x_1 with respect to its own price p_1, denoted ε_{x_1,p_1}, is such that $0 < \varepsilon_{x_1,p_1} < 1$ and x_1 is a normal good.

1. Write down the Lagrangian for the minimization of expenditure with utility held constant.

2. Find the first-order condition for expenditure minimization with respect to x_1 only (do not eliminate the Lagrange multiplier).

3. With the level of utility U held constant, how does a rise in b affect expenditure? (give the answer in terms of prices, quantities and the value of b).

4. With the level of utility U held constant, does a rise in b raise or lower the quantity of x_1 demanded? Does it raise or lower the consumer surplus gained from being able to buy x_1 at price p_1?

5. With the level of utility U held constant, consider the consumer surplus gained from being able to buy one of the other goods, x_k at price p_k, $k = 2, ..., L$. Is it possible for a rise in b to raise everyone of the consumer surpluses?

Exercise 4

Let us consider a consumer who wants to consume two goods, labelled 1 and 2, in quantities x_1 and x_2 respectively. The price vector is $p = (p_1, p_2)$ and the exogenous income amounts to w. The consumer's preferences are represented by the utility function

$$U(x_1, x_2) = \sqrt{x_1} + \sqrt{x_2}. \tag{7}$$

1. *The optimal choice of the consumer.*

   (a) Are the preferences represented by U convex?

   (b) Derive the Marshallian demands for good 1 and good 2, denoted $x_1(p, w)$ and $x_2(p, w)$ respectively. Examine how changes in prices and income alter the individual consumption behaviour.

   (c) Numerical application. Let $p_1 = 1$, $p_2 = 2$ and $w = 6$. What are the optimal Marshallian demands? These numerical values are used in question 2.

2. *Tax Revenue.* The policy maker decides to levy taxes, but hesitates between two tax mechanisms.

   (a) Indirect taxes. The policy maker implements a tax t on the price of good 1 (so that an amount $p_1 t$ is levied on each unit of good 1 bought by the consumer). It is herein assumed that $t = 1$. Compute how the optimal demands for good 1 and good 2 are altered w.r.t. the situation in which there is no taxes. Explain? Derive the tax revenue T obtained by the policy maker.

   (b) Income taxes. Now, the policy maker decides to obtain the same tax revenue T but uses an income tax instead of an indirect tax. Derive the changes in the optimal demand for good 1 and good 2 w.r.t. the tax-free situation. Explain?

   (c) Use a graph, in the (x_1, x_2)-space, to represent the optimal choices in the three cases considered above. What would you recommend?

Exercise 5

Consider a consumer whose expenditure function is $e(p, u)$. His demand function for good j is $x_j(p, w)$, where w is his income and p is the price vector. Show that good j is normal if and only if $\frac{\partial^2 e}{\partial p_j \partial u} > 0$.