Paris School of Economics, Master 1 APE
Microeconomics 1, Problem Set 6

Michaelmas Term 2007-2008

Exercise 1

Consider a Cobb-Douglas production function:

$$f(x) = x_1^{\alpha} x_2^{\beta}, \text{ with } \alpha > 0 \text{ and } \beta > 0. \tag{1}$$

Part I:

1. Sketch the production possibility set Z in each of the following cases:
   (a) $\alpha + \beta < 1$;
   (b) $\alpha + \beta > 1$;
   (c) $\alpha + \beta = 1$.
   Compute the marginal product, average product, and marginal rate of technical substitution, as function of x_1.
2. Is the production function homothethic and/or homogeneous?
3. What returns to scale are there?
4. Derive the factor demands $x_1(p,q)$ and $x_2(p,q)$.
5. Derive the supply function $y(p,w)$.
6. Find the marginal price effects. Confirm the signs (and where appropriate, relative magnitudes) of these effects.
7. Find the profit function.
8. Confirm that Hotelling's Lemma holds.

Part II:

1. Derive the conditional factor demand $h_1(w,q)$ and $h_2(w,q)$.
2. Find the cost function $c(w,y)$. Confirm is properties.
3. Sketch $c(w,y)$ as a function of y for each of the case $\alpha + \beta > 1$, $\alpha + \beta < 1$ and $\alpha + \beta = 1$. In addition, sketch marginal and average costs as a function of y for each of the three cases.
4. Why is it not necessary to assume that $\alpha + \beta < 1$ for cost minimization?
5. Confirm that Shepard's Lemma holds.

Exercise 2

Consider a two-good ($\ell = 1,2$), two-individual economy ($i = A,B$). Individual preferences are represented by the utility functions,

$$U^A(x) = \frac{1}{3}\ln x_1 + \frac{2}{3}\ln x_2 \text{ for } A,$$

and

$$U^B(x) = \frac{3}{4}\ln x_1 + \frac{1}{4}\ln x_2 \text{ for } B.$$

The initial endowments in goods 1 and 2, respectively, are $e^A = (3,9)$ for A and $e^B = (8,12)$ for B.

1. Characterize the set of Pareto optimal allocations.
2. Derive the Walrasian general equilibrium.
3. Check that the Walrasian equilibrium is Pareto optimal. Comment.
4. Check that the allocation $x_1 = \left(x_1^A, x_1^B\right) = (5.5, 18)$, $x_2 = \left(x_2^A, x_2^B\right) = (5.5, 3)$ is Pareto optimal. Compute the transfers required to decentralized it as a competitive equilibrium. Comment.

   A firm which transforms good 1 into good 2 is now added to the previous economy. The production technology is summarized by

   $$y_2 = 4y_1, \text{ with } y_1 \geq 0.$$

5. Compute the Walrasian equilibrium.
6. Assume there are two states of nature, s_1 and s_2. Use the theory of individual decision making under uncertainty to reinterpret the model presented in this exercise.

Exercise 3

In an economy without any uncertainty, there are two commodities, education (e) and food (f), produced by using labour L and land T according to the production functions

$$e = (\min\{L,T\})^2 \text{ and } f = (LT)^{1/2}.$$

There is a single consumer with the utility function

$$u(e,f) = (e)^{\alpha}(f)^{1-\alpha},$$

and endowment (ω_L, ω_T). To ease the calculations, $\omega_L = \omega_T = 1$ and $\alpha = 1/2$. What is the set of Pareto optimal allocations?

Exercise 4

Consider a two-good, two-individual economy. The preferences of individual A and individual B over good 1 and good 2 are represented by the utility functions:

$$u^A(x) = x_1 + x_2 \text{ and } u^B(x) = x_1 + 2x_2, \tag{2}$$

respectively. The initial endowments are:

$$e^A = (1,2) \text{ and } e^B = (2,1). \tag{3}$$

Determine the Walrasian equilibria.

Exercise 5

Consider a two-individual, three-good economy. The preferences of individual A and individual B over the consumption bundles $x = (x_1, x_2, x_3) \geq 0$ are represented by the utility functions:

$$u^A(x) = x_1 x_2^m x_3^n \tag{4}$$
$$u^B(x) = (x_1 + k) x_2^m x_3^n, \; m \geq 0, n \geq 0, \; k \geq 0. \tag{5}$$

respectively. The initial endowments are:

$$e^A = e^B = (1, 1, 1). \tag{6}$$

Determine the Walrasian equilibria as the preference parameter k varies.

Exercise 6

Consider a two-good, two-individual economy. The preferences of individual A and individual B over good 1 and good 2 are represented by the utility functions:

$$u^A(x) = x_1 \text{ and } u^B(x) = x_1 + \frac{1}{\alpha} x_2^{\alpha}, \; x_1 \geq 0, x_2 \geq 0, 0 < \alpha < 1 \tag{7}$$

respectively. The initial endowments are:

$$e^A = (1, k), \; k > 0, \text{ and } e^B = (1, 1). \tag{8}$$

Determine the Walrasian equilibria as the endowment parameter k and the preference parameter α vary.