

Microeconomics 1, Master 1 APE

Exam

Michaelmas Term 2006-2007

Briefly answer the following questions:

1. Prove that a Giffen good is necessarily an inferior good.
2. What is the link between a firm's supply and its marginal cost? (Draw a graph)
3. State Hotelling's lemma.

Exercise 1

Forecasters predict there is a 50% probability that the upcoming growing season will be a drought. Assume Ted is an expected utility maximizer with a Bernoulli function $u(x) = \ln(x)$. Ted's initial wealth is equal to zero.

Ted initially has the choice between two crops (Potatoes/Strawberry) yielding the following payoffs (in euros):

	Normal Rain	Drought
Potatoes	5 000	40 000
Strawberry	20 000	12 000

There are constant returns to scale in (the payoffs of) each production.

1. If Ted can only plant one crop, which crop should he plant?
2. Ted can choose any combination of Potatoes and Strawberry crops. Which crop mix should he choose?
3. Assume Ted decides to plant half of his land with each crop. He is offered Strawberry insurance. This insurance costs 5 000 euros and pays 10 000 euros in the case of a drought. Should Ted buy it?

Exercise 2

Let \succsim be Mister A 's preference relation over consumption bundles $(x_1, x_2) \in \mathbb{R}^+ \times \mathbb{R}^+$, where x_i is the quantity of good i ($i = 1, 2$). \succsim is represented by the following utility function:

$$U(x_1, x_2) = x_1 + \sqrt{x_2}.$$

Mister A 's exogenous wealth is w^A and $p_i > 0$ is the price of good i , $i = 1, 2$.

1. Draw a few indifference curves and explain why corner solutions cannot be ruled out.

2. Derive carefully Mister A 's Marshallian demand functions.
3. Draw the Engel curves corresponding to good 1 and good 2 (use $p_1 = 2$ and $p_2 = 1$ to draw the graph). Comment the income effects. Then, examine and comment the direct and indirect price effects.
4. Compute Mister A 's indirect utility and expenditure functions.
5. Derive Mister A 's Hicksian demand functions.
6. Use the Slutsky equation to confirm that all income effects are absorbed by good 1 when Mister A 's wealth w^A allows him to consume a positive amount of each good.

Exercise 3

The supply side of an economy consists of 8 (price-taker) identical producers. Each producer is endowed with a technology which uses two inputs (x_1 and x_2) and produces one output y . This technology is described by the following cost function:

$$c(\omega_1, \omega_2, y) = \left(\beta \omega_1^\beta \omega_2^\beta \right) y^4, \quad (1)$$

where ω_1 and ω_2 are the input prices. We assume $\beta > 0$.

1. Show that β must be equal to $1/2$ for $c(\omega_1, \omega_2, y)$ to satisfy the properties of a cost function.
From now on, we take: $\beta = 1/2$.
2. Find the production function.
3. Identify the returns to scale and represent the production possibility set of a producer.
4. What is the aggregate supply function, denoted Y , of commodity y ?

The demand side of this economy consists of 8 (price-taker) identical consumers. Each consumer consumes only the output commodity y and has preferences represented by the strictly increasing function $u(y)$. In addition, each consumer's exogenous income is equal to 2.

5. Identify each consumer's Marshallian demand for good y . Deduce the consumers' aggregate demand in this market (i.e. the sum of the 8 Marshallian demand functions).
6. Derive the equilibrium price and quantity in this market.

Exercise 4

Consider a two-good (1 and 2), two-individual (A and B) economy. Let $x = (x_1, x_2) \in \mathbb{R}^+ \times \mathbb{R}^+$ be a consumption bundle. Mister A and Mister B have identical preferences, represented by the utility function defined over consumption bundles, $u : \mathbb{R}^+ \times \mathbb{R}^+ \rightarrow \mathbb{R}$ with

$$u(x) = x_1 x_2.$$

Mister A and B 's endowments in goods 1 and 2 are $\omega^A = (3, 0)$ and $\omega^B = (0, 3)$ respectively. The price of good i is denoted $p_i > 0$ ($i = 1, 2$).

1. Derive the set of Pareto optimal allocations of this economy. Represent it in the Edgeworth box.
2. Compute the Walrasian general equilibrium. Comment.
3. A benevolent policy maker is introduced in the economy. Determine all transfers he can choose if he wants to decentralize the allocation $(x_1^A, x_2^A, x_1^B, x_2^B) = (1, 1, 2, 2)$. Comment.
4. A productive sector is now added to the pure exchange economy considered above. The productive sector consists of one firm which transforms good 1 in good 2 according to the production function:

$$y_2 = f(y_1) = ay_1, \text{ where } a > 0.$$

Compute the Walrasian general equilibrium of this economy. Comment.