Microeconomics 1, Master 1 APE
Exam 2

22 January 2007

Please write legibly and show your work clearly.

Briefly answer the following questions: (You should not write more than 10 lines per answer)

1. Explain the first theorem of welfare economics.
2. Write down and comment Slutsky equation.
3. Under which conditions a preference relation is said to be rational?
4. State Shepard's lemma.

Exercise 1

Fred is an expected utility-maximizing driver who likes to speed. He can either exert self-restraint (at a high effort) and therefore drive safely, or self-indulgence (with low effort) and drive recklessly. Let high effort be represented by $e = 1$ and low effort by $e = 0$. Suppose further that Fred's preferences over effort and wealth are represented by the utility function

$$u(w,e) = \sqrt{w} - 5e.$$

Suppose Fred has an *initial wealth of 1000 euros*. If Fred gets in an accident, he incurs a *500-euro loss*. The probability that Fred gets in an accident depends on his effort level:

$$\begin{aligned} \text{if } e &= 0,\ \Pr\{\text{accident}\} = \frac{2}{3}, \\ \text{if } e &= 1,\ \Pr\{\text{accident}\} = \frac{1}{4}. \end{aligned}$$

Assume there are no other costs associated with the accident besides the 500 euros.

1. Will Fred drive safely or recklessly?

From now on, it is assumed that Fred drives recklessly, i.e. $e = 0$ and $\Pr\{\text{accident}\} = \frac{2}{3}$. An insurance company designs an insurance contract: Fred would pay an insurance premium of 200 euros to receive 400 euros in the event of an accident.

2. Does Fred decide to buy this insurance contract?
3. What is the minimum amount of money the insurance company must give to Fred in the event of an accident if it wants him to buy the insurance contract? (the insurance premium is unaltered)

Exercise 2

Consider a firm with productivity function

$$f(x) = 20x - x^2.$$

Normalize the price of the output good to one and let the relative price of the input good, x, be ω. Only non-negative amounts of input good can be used.

1. Write down the firm's optimization programme and derive its first-order condition.
2. For which values of ω will the optimal value of x be zero?
3. For which values of ω will the optimal x be 10?
4. Find the conditional factor demand as a function of ω.
5. Find the profit function.

Exercise 3

A consumer has the following utility over childcare c and food f :

$$u(c,f) = c^{\frac{1}{5}} f^{\frac{4}{5}}.$$

The price of childcare is $p_c = 2$, the price of food is $p_f = 4$ and the consumer's income is $I = 20$.

1. What are the consumer's (Marshallian) demands for childcare and food?
2. Compute the consumer's expenditure function, $e\left(p_c, p_f, u\right)$.
3. Derive the consumer's Hicksian demands for childcare and food.

The policy maker decides to implement a redistributive policy. He has the choice between an income subsidy and an in-kind transfer scheme. He is fortunate enough to know you; so he decides to mobilize your microeconomic skills to help him.

4. Suppose first that the government gives the consumer an income subsidy of $S = 10$. How does it alter the consumer's (Marshallian) demands for childcare and food?
5. Suppose now that the government decides to replace the income subsidy by an in-kind transfer to the consumer. The in-kind transfer takes the form of 4 hours of childcare and 0.5 units of food (It is assumed that this transfer cannot be re-sold by the consumer). Formally, this means that the consumer is constrained to choose $c \geq 4$ and $f \geq 0.5$, but only pays for the units consumed above these thresholds.
   (a) Write down the consumer's optimization programme. Deduce that the consumer's optimal demand for childcare is $c = 4$. Then, compute his optimal demand for food f.
   (b) Suppose the consumer must buy all goods on the market. What is the minimum expenditure level required to attain the same utility level as under the in-kind transfer? (You can use your answer to Question 2)
6. Which redistributive policy would you recommend to the government? The cash subsidy or the in-kind transfer? Comment.

Exercise 4

Consider a two-good (1 and 2), two-individual (A and B) economy. Let $x = (x_1, x_2) \in \mathbb{R}^+ \times \mathbb{R}^+$ be a consumption bundle. Mister A's preferences are represented by the utility function $u_A : \mathbb{R}^+ \times \mathbb{R}^+ \longrightarrow \mathbb{R}$, with

$$u_A(x) = \min\{x_1, x_2\},$$

while Mister B's preferences are represented by the utility function $u_B : \mathbb{R}^+ \times \mathbb{R}^{++} \longrightarrow \mathbb{R}$, with

$$u_B(x) = x_1 + \ln x_2.$$

Mister A and B's endowments in goods 1 and 2 are $\omega^A = (3,0)$ and $\omega^B = (0,3)$ respectively. The price of good i is denoted $p_i > 0$ ($i = 1,2$).

1. Represent this economy in the Edgeworth box. In particular, sketch a few indifference curves and represent the initial endowments.
2. What is the set of Pareto optimal allocations of this economy? Represent it in the Edgeworth box.
3. Compute the Walrasian general equilibrium. Check that it belongs to the Pareto set. Comment.
4. A productive sector is now added to the pure exchange economy considered above. The productive sector consists of one firm which transforms good 1 in good 2 according to the production function:

   $$y_2 = \sqrt{y_1},$$

   where y_1 is the quantity of input used and y_2 the quantity of output produced by the firm.

   (a) Write down and solve the firm's programme.

   (b) Assume the profits of the firm are redistributed equally among consumers. Compute the Walrasian general equilibrium of this economy. (Hint: The unique positive root of $3 + 28p_1 - 8p_1^2 - 8p_1^3$ is $p = 3/2$.)