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Chapter 2 - Consumer choice (in the spirit of the choice-based approach)

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Introduction

Definition (Market)

The "place" where demand and supply meet.

A setting in which consumers can buy products at known prices (or, equivalently, trade goods at known exchange rates).

Question: How do consumers make constrained choices?

Basic concepts

Goods (and services)

- Finite number L of divisible goods (commodities)
- $\mathbb{R}_+^L$ is the *commodity space*
- $X \subset \mathbb{R}_+^L$ is the *consumption set*
- $x \in X$ is a consumption vector or *consumption bundle*

Consumption set

Elements of X are bundles that a individual may consume given the context's *physical constraints*.

Examples

Consumption of bread and leisure: $X = \left\{(b, l) \in \mathbb{R}^2_+ : l \leq 24\right\}$

Minimum consumption of white or brown bread (survival consumption): $X = \left\{(w, b) \in \mathbb{R}^2_+ : w + b \geq 4\right\}$

$X = \mathbb{R}^L_+ = \left\{x \in \mathbb{R}^L : x_l \geq 0,\ l = 1 \ldots L\right\}$

Budget set

The price vector $p = (p_1, \ldots p_L)$ gives the price associated with each good $i = 1, \ldots L$.

Definition (Feasibility)

When income or wealth is w, the consumption bundle x is feasible (affordable) if:

$$p \cdot x = (p_1, \ldots p_L)^T (x_1, \ldots x_L) = \sum_{i=1}^{L} p_i x_i \leq w$$

Definition (Budget set)

The budget set is the set of all feasible consumption bundles for the consumer who faces market prices p and has wealth w:

$$B(p, w) = \left\{ x \in \mathbb{R}_+^L : p \cdot x \leq w \right\}$$

Definition (Bugdget hyperplane)

A budget hyperplane is the set:

$$\left\{ x \in \mathbb{R}_+^L : p \cdot x = w \right\}$$

If $L = 2$, it is a line called the *budget constraint*.

Example. Graph. Prices are orthogonal to the *b.c.*

The demand function

Definition (Demand function)

A consumer facing a price-wealth pair (p, w) chooses a consumption bundle $x(p, w)$ (the demand correspondence or demand function if single-valued).

- In chapter 3, the demand is derived by maximizing utility s.t. a budget constraint.
- What properties should be expected?

We make two assumptions regarding the demand function:

Definition (Homogeneity of degree 0)

A demand function $x(p, w)$ is homogeneous of degree 0 if

$$x(\alpha p, \alpha w) = x(p, w) \text{ for any } (p, w) \text{ and } \alpha > 0$$

.

Definition (Satisfaction of Walras' law)

A demand function $x(p, w)$ satisfies Walras' law if

$$\text{for every } p \gg 0) \text{ and } w > 0, \text{ we have } p \cdot x(p, w) = w$$

i.e. the budget constraint is binding.

- **Remark**:

  $\mathcal{B} = \{B(p, w) : p \gg 0 \text{ and } w > 0\}$
  is a family of non-empty subsets of X.

  $x(p, w)$
  only depends on the budget set the consumer is facing.

  $\Rightarrow$ $(\mathcal{B}, x(.,.))$ is a choice structure.

- How does x vary with p and w?

Comparative statics

Wealth effects

Consider $\bar{p}$ a vector of fixed prices.

Definition (Engel function)

The function $x(\bar{p}, w)$ is called the Engel function.

Definition (Wealth expansion path)

$E_{\bar{p}} = \{x(\bar{p}, w) : w > 0\}$ *is the wealth expansion path.*

Graph

$\frac{\partial x_l(p,w)}{\partial w}$ measures the *wealth effect* for good $l = 1, \ldots L$

Definition (Normal vs inferior goods)

$\frac{\partial x_l(p,w)}{\partial w} \geq 0 \ \Leftrightarrow$ *the good l is* ***normal*** *for (p,w)*

$\frac{\partial x_l(p,w)}{\partial w} < 0 \ \Leftrightarrow$ *the good l is* ***inferior*** *for (p,w)*

Examples

Price effects

$\frac{\partial x_l(p,w)}{\partial p_k}$ measures the *price effect* of p_k, the price of good k, on the demand for good l (general case, cross derivative).

Consider first: $k = l$

Definition (Giffen good)

In most cases, $\frac{\partial x_l(p,w)}{\partial p_l} \leq 0$

$\frac{\partial x_l(p,w)}{\partial p_l} > 0 \Leftrightarrow$ *l is a Giffen good at (p,w)*

Example. Discussion of Giffen and Veblen effect.
Only inferior goods can be giffen goods (see chapter 3).

Definition (Elasticity)

The elasticity of demand for good l with respect to the price of good k is

$$\varepsilon_{l,k}(p,w) = \frac{\partial x_l(p,w)}{\partial p_k}\frac{p_k}{x_l(p,w)}$$

The wealth(or income)-elasticity of demand for good l is

$$\varepsilon_{l,w}(p,w) = \frac{\partial x_l(p,w)}{\partial w}\frac{w}{x_l(p,w)}$$

Comment: Elasticities give the percentage change in demand for good l per (marginal) percentage change in the price of good k or wealth.

Implications of homogeneity and Walras' law

(when prices and wealth change)

Proposition (Implication of homogeneity)

$x(p,w)$ homogeneous of degree 0

$$\Rightarrow \forall p, w \;\; \sum_{k=1}^{L} \frac{\partial x_l(p,w)}{\partial p_k} p_k + \frac{\partial x_l(p,w)}{\partial w} w = 0 \;\; \forall l = 1, \ldots L$$

which also writes

$$\forall p, w \;\; \sum_{k=1}^{L} \varepsilon_{l,k}(p,w) + \varepsilon_{l,w}(p,w) = 0 \;\; \forall l = 1, \ldots L$$

Proof.

Comment: An equal percentage change in all prices and wealth leads to no change in demand.

Proposition (Implication of Walras' law: Cournot Aggregation)

$x(p, w)$ satisfies Walras' law

$$\Rightarrow \ \forall p, w \ \ \sum_{l=1}^{L} p_l \frac{\partial x_l(p,w)}{\partial p_k} + x_k(p, w) = 0 \ \ \forall k = 1, \dots L$$

Comment: Total expenditure cannot change in response to a variation in prices.

The second term is the direct effect on expense resulting from a marginal increase in p_k.

The first term is the change in expenditure associated with the adjustment of the consumption bundle.

Proof.

Proposition (Implication of Walras' law: Engel Aggregation)

$x(p, w)$ satisfies Walras' law

$$\Rightarrow \; \forall p, w \;\; \sum_{l=1}^{L} p_l \frac{\partial x_l(p,w)}{\partial w} = 1$$

Comment: Total expenditure must change by an amount equal to any wealth change.

Proof.

The WA and law of (compensated) demand

- We assume the following:
  (i) Weak Axiom of Revealed Preferences (Chapter 1)
  (ii) Homogeneity of degree 0
  (iii) Walras' law
- i.e. we impose more consistency on choices. In fact, these three assumptions will be satisfied when we derive the consumer's demand from the classical demand theory (see the preference-based approach, next chapter).
- What are the implications?

Definition (Weak Axiom (comparing two situations))

The demand function $x(p, w)$ satisfies the WA if $\forall(p, w)$ and $\forall(p', w')$ we have the following property:

$$\text{If } p \cdot x(p', w') \leq w \text{ and } x(p', w') \neq x(p, w) \Rightarrow p' \cdot x(p, w) > w'$$

Intuition: If the bundle $x(p', w')$ is feasible when the agent faces price-wealth (p, w) and (by definition) the agent chooses $x(p, w)$, this **reveals** a preference of the agent for $x(p, w)$ over $x(p', w')$. **Then**, since the agent chooses $x(p', w')$ when facing price-wealth (p', w'), it **must be** that he cannot afford $x(p, w)$.

Proposition

Assuming the WA, one ***cannot observe***

$$p \cdot x(p', w') \leq w \text{ and } p' \cdot x(p, w) \leq w'$$

$$\text{with } x(p, w) \neq x(p', w')$$

Examples.

Implications of the WA

- One important issue: the effect of a **change in prices**. The effect is ambiguous. A decrease in a price can decrease consumption (Giffen good) when the wealth effect dominates the price effect. **Explain.**
- However, we can neutralize the wealth effect. Assuming the WA, what implications do we get?

Definition (Slutsky wealth compensation)

Slutsky wealth compensation is a situation in which a change in prices ($\Delta p = p' - p$) is accompanied by a change in the consumer's wealth ($\Delta w = w' - w$) that makes his initial consumption bundle $x(p, w)$ just affordable at the new prices.

$$p \cdot x(p, w) = w$$

$$p' \cdot x(p, w) = w'$$

$$\Delta w = \Delta p \cdot x(p, w)$$

Graph.

Proposition (Law of (compensated) demand)

Assume that $x(p, w)$ is homogeneous of degree 0, and satisfies Walras' law:

$x(p, w)$ satisfies the WA

$$\Leftrightarrow$$

For any (Slutsky) compensated variation of prices (i.e. from (p, w) to (p', w') with $w' - w$ that compensates the price variation), we have:

$$(p' - p) \cdot \; [x(p', w') - x(p, w)] \leq 0$$

Proposition (Implication of the law of (compensated) demand)

Under our three assumptions, the ***compensated demand*** *for a good decreases in its own price:*

$$\frac{\partial x_l(p, w)}{\partial p_l} \leq 0$$

Proof. Graph.

Conclusion

- We started building a theory of demand under three assumptions (consistency requirement). Can we infer a rational structure from there?
- **No**. Example of a demand satisfying the WA but the revealed preferences violate transitivity (and thus cannot be rational).
- $\Rightarrow$ The preference-based approach