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Chapter 6 - An introduction to general equilibrium

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Definition (Competitive equilibirum)

An allocation $(x^{1*}, ..., x^{N*}, y^{1*}, .., y^{Q*})$ *and price vector* $p^* \in R_+^M$ *is a* ***competitive (or Walrassian) equilibrium*** *if the following conditions are satisfied:*

- *Firms maximize their profit*
- *Consumers maximize their utilities*
- *The market for each good clears (i.e. consumption and resources are equalized)*

A simple model (from Mas-Collel, p.525)

Notations:

- One consumer, one firm, two “goods” (leisure, and the good produced by the firm)
- z is the labor input
- $f(.)$ is the strictly-concave production function
- p is the price of the output
- w is the price of labor

The producer's program

$$\underset{z \geq 0}{\text{Max}}\ p.f(z) - w.z$$

Solving the program yields:

- a labor demand $z(p, w)$
- a product supply $q(p, w)$
- a profit function $\Pi(p, w)$

The consumer's program

$$\begin{cases} \underset{x_1, x_2 \geq 0}{\text{Max}} \; u(\underbrace{x_1}_{\text{leisure}}, \underbrace{x_2}_{\text{good}}) \\ \text{s.t.} \quad \begin{array}{l} p.x_2 \leq w.(\underbrace{\overline{L} - x_1}_{\text{hours worked}}) + \Pi(p, w) \\ \overline{L} \geq x_1 \end{array} \end{cases}$$

Solving the program yields:

- a demand for leisure $x_1(p, w)$
- a demand for the good $x_2(p, w)$

Defining an equilibrium

- An equilibrium is a situation in which supplies and demands are equalized. It involves a price vector (p^*, w^*) such that the consumption and labor markets clear:

$$\begin{aligned} \text{on the good market} &: \quad x_2(p^*, w^*) = q(p^*, w^*) \\ \text{on the labor market} &: \quad z(p^*, w^*) = \overline{L} - x_1(p^*, w^*) \end{aligned}$$

- It is associated with a vector of endogenous quantities and prices:

$$\mathcal{E} = \{p^*, w^*, x_1^*, x_2^*, z^*, q^*, \Pi^*\}$$

Equilibrium and optimum

Under a set of assumptions (and in particular in the absence of "externality"), we have the two fundamental theorems of welfare economics:

Proposition (1st theorem)

The first fundamental theorem of welfare economics:

*Any **competitive equilibrium** is a **Pareto optimum**.*

Proposition (2nd theorem)

The second fundamental theorem of welfare economics:

*Any **Pareto optimum** can be "decentralized" as a **competitive equilibrium** (i.e. by changing initial endowments and letting the market function).*

Example

- Two consumers (1 and 2), each offering a quantity *n* of work.
- Two firms (1 and 2).
- Profits are redistributed equally among consumers (who are also shareholders).

$$\begin{cases} u_1(x_1^1, x_2^1) = \frac{1}{3}\ln x_1^1 + \frac{2}{3}\ln x_2^1 \\ u_2(x_1^2, x_2^2) = \frac{2}{3}\ln x_1^2 + \frac{1}{3}\ln x_2^2 \\ f_1(l) = \sqrt{l} \\ f_2(l) = 2\sqrt{l} \end{cases}$$