Definition (Competitive equilibrium)

An allocation \((x^{1*}, ..., x^{N*}, y^{1*}, ..., y^{Q*})\) and price vector \(p^* \in \mathbb{R}_+^M\) is a competitive (or Walrasian) equilibrium if the following conditions are satisfied:

- **Firms maximize their profit**
- **Consumers maximize their utilities**
- **The market for each good clears (i.e. consumption and resources are equalized)**
Notations:

- One consumer, one firm, two “goods” (leisure, and the good produced by the firm)
- $z$ is the labor input
- $f(.)$ is the strictly-concave production function
- $p$ is the price of the output
- $w$ is the price of labor
The producer’s program

$$\text{Max } p.f(z) - w.z$$

Solving the program yields:

- a labor demand $z(p, w)$
- a product supply $q(p, w)$
- a profit function $\Pi(p, w)$
The consumer’s program

\[
\begin{align*}
\text{Max} & \quad u(x_1, x_2) \\
\text{s.t.} & \quad \bar{L} \geq x_1 \\
\end{align*}
\]

\[
\begin{align*}
& x_1, x_2 \geq 0 \\
& p.x_2 \leq w.(\bar{L} - x_1) + \Pi(p, w) \\
\end{align*}
\]

Solving the program yields:

- a demand for leisure \( x_1(p, w) \)
- a demand for the good \( x_2(p, w) \)
Defining an equilibrium

- An equilibrium is a situation in which supplies and demands are equalized. It involves a price vector \((p^*, w^*)\) such that the consumption and labor markets clear:

  on the good market : \(x_2(p^*, w^*) = q(p^*, w^*)\)
  on the labor market : \(z(p^*, w^*) = \bar{L} - x_1(p^*, w^*)\)

- It is associated with a vector of endogenous quantities and prices:

  \[\mathcal{E} = \{p^*, w^*, x_1^*, x_2^*, z^*, q^*, \Pi^*\}\]
Under a set of assumptions (and in particular in the absence of “externality”), we have the two fundamental theorems of welfare economics:

**Proposition (1st theorem)**

*The first fundamental theorem of welfare economics:*

Any *competitive equilibrium* is a *Pareto optimum*. 
Proposition (2nd theorem)

*The second fundamental theorem of welfare economics:*

Any *Pareto optimum* can be “decentralized” as a *competitive equilibrium* (i.e. by changing initial endowments and letting the market function).
Example

- Two consumers (1 and 2), each offering a quantity $n$ of work.
- Two firms (1 and 2).
- Profits are redistributed equally among consumers (who are also shareholders).

\[
\begin{align*}
  u_1(x_1^1, x_2^1) &= \frac{1}{3} \ln x_1^1 + \frac{2}{3} \ln x_2^1 \\
  u_2(x_1^2, x_2^2) &= \frac{2}{3} \ln x_1^2 + \frac{1}{3} \ln x_2^2 \\
  f_1(l) &= \sqrt{l} \\
  f_2(l) &= 2\sqrt{l}
\end{align*}
\]